University of Swaziland

Final Examination – May 2009

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations
Course Number : M213
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

Find the general solution of each differential equation.

(a) $y'' + 4y' - 21y = 0$ [4 marks]

(b) $y^{iv} + 5y'' - 36y = 0$ [4 marks]

(c) $y(1 + 2xy)\mathrm{d}x + x(1 - xy)\mathrm{d}y = 0.$ [12 marks]

Question 2

(a) Find the inverse Laplace transform of

$$F(s) = \tan^{-1}\left(\frac{s}{a}\right).$$ [8 marks]

(b) Find the particular solution of

$$y(x+y^3)\mathrm{d}x + x(y^3 - x)\mathrm{d}y = 0,\ y(2) = -1.$$ [12 marks]

Question 3

Find the series solution of

$$xy'' + (1 - 2x)y' + (x - 1)y = 0$$

about $x = 0$. [20 marks]

Question 4

(a) Use Laplace transforms to solve

$$\ddot{y} + 4y = t,\ \ y(0) = 1,\ \dot{y}(0) = 0.$$ [14 marks]

(b) Solve

$$x(1 + y^2)\mathrm{d}x - y^2(1 - x^2)\mathrm{d}y = 0.$$ [6 marks]

Question 5

(a) Solve the boundary-value problem

$$y''-4y'+8y = \sin x, \quad y(0) = 1, \quad y'(\pi) = 0. \quad \text{[14 marks]}$$

(b) Find the general solution of

$$2x^2y'' - 3xy' + 2y = 0. \quad \text{[6 marks]}$$

Question 6

(a) Use *two methods* to find the particular integral, and hence general solution of the equation

$$y'' - 4y = e^{-2x}. \quad \text{[14 marks]}$$

(b) Solve the ordinary differential equation

$$y'' - 2y' = 1. \quad \text{[6 marks]}$$

Question 7

Solve

(a) $(1 + x^2)y' + x + x^3 = xy$ [10 marks]

(b) $xy' = y(1 + \ln x - \ln y), \quad y(1) = e^{1-\pi}.$ [10 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{\mathrm{d}^n f}{\mathrm{d}t^n}(t)$	$s^n F(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$