University of Swaziland

Supplementary Examination – July 2009

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations
Course Number : M213
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

Find the general solution of each differential equation.

(a) $y'' + 4y' + 4y = 0$ [4 marks]

(b) $y^{iv} - 16y = 0$ [4 marks]

(c) $\mathrm{d}y = (2y + x^2)\mathrm{d}x.$ [12 marks]

Question 2

(a) Find the inverse Laplace transform of

$$F(s) = \frac{4s + 4}{s^4 + 4s^2}.$$ [12 marks]

(b) Find the general solution of

$$x(1 + y)\mathrm{d}x - y(1 - x)\mathrm{d}y = 0, \quad y(-1) = 1.$$ [8 marks]

Question 3

Find the series solution of

$$x^2y'' + xy' - 4y = 0$$

about $x = 0$. [20 marks]

Question 4

(a) Use Laplace transforms to solve

$$\ddot{y} + 4y = 0, \quad y(0) = 1, \ \dot{y}(\pi) = 1.$$ [10 marks]

(b) Solve

$$(y + \ln x)\mathrm{d}x - x\mathrm{d}y = 0.$$ [10 marks]

Question 5

(a) Solve the boundary-value problem

$$y'' + 2y' - 15y = 30e^{-2x}, \quad y(0) = -2, \; y'(0) = -4.$$

[14 marks]

(b) Find the general solution of

$$x^2y'' + 2xy' = 1.$$ [6 marks]

Question 6

(a) Find the general solution of the equation

$$y'' + 4y = \sin 2x,$$

using the method of undetermined coefficients.

[14 marks]

(b) Using the problem in (a), write a concise *description* of the method of variation of parameters. [6 marks]

Question 7

Solve

(a) Write a short essay on: [6 marks]

exact equations and integrating factors.

(b) Solve

$$(2x-5y+14)\,dx-(2x+4y-4)\,dy = 0.$$ [14 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(at)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(at)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{\mathrm{d}^n f}{\mathrm{d}t^n}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$