UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2008/09

BSc.II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE (5) QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY TEH INVIGILATOR.

QUESTION 3

(a)Let A, B and C be (2×2) matrices. Determine which of the following is true. Give examples.

(i) $AB = O \Rightarrow A = 0$ or $B = 0$,

(ii) $AB = AC \Rightarrow A = 0$ or $B = C$. [2,2]

(b) Consider the matrix

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & -6 \\ 2 & -1 & 3 \end{bmatrix}.$$

(i) Find the minors and the cofactors for each entry.

(ii) Expand, simplify and evaluate the determinant.

(iii) Check your solutin by applying a direct formula for the determinant of (3×3) matrix. [4,2,3]

(c) Apply Lagrange's method to find min $f(X)$

$$f(X) = 2x_1^2 + x_2^2$$

subject to the constraint

$$x_1 + 2x_2 = 3.$$

[7]

QUESTION 4

(a) Find the inverse and check the result, or state that the inverse does not exist, giving reasons

(i) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

(ii) $\begin{bmatrix} 6 & -2 & \frac{1}{2} \\ 1 & 5 & 2 \\ -8 & 24 & 7 \end{bmatrix}$ [2,4]

(b) Solve the following system

$$\begin{aligned} 3x_1 + 2x_2 + x_3 &= 3 \\ 2x_1 + x_2 + x_3 &= 0 \\ 6x_1 + 2x_2 + 4x_3 &= 6 \end{aligned}$$

applying

(i) Gauss elimination method,

(ii) Cramer's rule. [4,4]

(c) Let D be the region defined by the inequalities

$$x^2 + y^2 < 2, \quad 0 < z < x^2 + y^2.$$

Pass to the cylindrical coordinates to find

$$\iiint_D x^2y^2\,dx\,dy\,dz$$

[6]

QUESTION 5

(a) Find the equation of the line through $(6, -3)$ which is

(i) parallel,

(ii) perpendicular

to the line $x - 3y + 12 = 0$. [3,3]

(b) State L'Hospial rule. [3]

(c) Apply the above rule to evaluate the following limits

(i) $\lim\limits_{x \to 0} \dfrac{\tan x}{x}$,

(ii) $\lim\limits_{x \to +\infty} \dfrac{x^2 + 3x + 1}{4x^2 - 5x + 1}$. [3,3]

(d) A ladder 20m long learns against a vertical wall. The bottom of the ladder is pulled away from the wall at the rate of 8m/min. How fast is the top of the ladder moving down the wall when the bottom of the ladder is 12m from the wall? [5]

QUESTION 6

(a) State and prove the mean value theorem (MVT). [5]

(b) Let $f(x) = \frac{4}{x}$. Find all numbers in the open interval $(1, 4)$ which satisfy the MVT on the interval $[1, 4]$. [3]

(c) Find the area enclosed between the parabola $y = x^2$, the y-axis and the tangent to this parabola at the point $(1, 1)$. [5]

(d) Find the surface area of a sphere of radius R. [7]

QUESTION 7

(a) Find the partial derivatives (at $x = 0, \quad y = \pi$) of $f(x, y) = \sin(x^2 + y)$. [5]

(b) Suppose that $F(x, y) = 3x^2y, \quad x = \varphi(u, v) = u + v$ and $y = \psi(u, v) = uv$. Set $z = F(\varphi(uv), \psi(u, v))$ and find the partial derivatives $\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}$ at $u = 2, \quad v = 3$.

[7]

(c) Compute the double integral

$$\int_0^{\frac{\pi}{2}} \int_0^{\pi} \sin x \quad \cos y \ dx \ dy.$$

[8]

END OF EXAMINATION