UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008/9

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1.1 Prove that if $A \Rightarrow B$, $B \Rightarrow C$, and $C \Rightarrow A$, then A is equivalent to B and A is equivalent to C. [10]

1.2 Determine the following sets:

(a) $\{m \in \mathbb{N} : \exists n \in \mathbb{N} \text{ with } m \leq n\}$; [3]

(b) $\{m \in \mathbb{N} : \forall n \in \mathbb{N} \text{ we have } m \leq n\}$. [2]

1.3 Let a be an algebraic number and let r be a rational number. Show that $a - r$ is an algebraic number. [5]

QUESTION 2

2.1 Write down symbolically, the negation of the statements:

(a) $\exists x, (\neg P(x) \vee Q(x))$; [4]

(b) $\forall x \in \mathbb{R} \, \forall y \in \mathbb{R} \, \exists z \in \mathbb{R}, x^2 + y^2 < z$. [6]

2.2 Let $A = \{-3, -2, -1, 0, 1, 2, 3\}$, where $A \subseteq \mathbb{R}$. Determine the truth set of

$$(\forall y \in A), x + y < 5.$$

[5]

2.3 Determine the truth value in $\mathbb{R}$ of:

(a) $\exists x \in \mathbb{R}$ such that $|x| = -x$; [2]

(b) $\exists x \in \mathbb{R} : x + 4 = x$. [3]

QUESTION 3

3.1 State the difference between deductive reasoning and inductive reasoning. Which of the two is a valid form of argument? Explain. [4]

3.2 Prove that if n is an integer and n^2 is divisible by 2, then so is n. [6]

3.3 Define the following:

(i) Fallacy of affirming the conclusion; [2]

(ii) Fallacy of denying the antecedent. [2]

3.4 Using truth tables, analyze the following argument and state whether it is valid or invalid

"All Germans are Europeans.
My neighbor is not a German.
Therefore my neighbor is not a European." [6]

QUESTION 4

4.1 Describe a modified induction procedure that could be used to prove statements of the form:

(a) For all integers $n \leq k$, $P(n)$ is true, where $P(n)$ is a statement containing the integer n. [3]

(b) For all integers n, $P(n)$, where $P(n)$ is as in (a). [4]

(c) For every positive odd integer, something happens. [3]

4.2 For all non-negative integers n define the number u_n inductively as

$$\begin{aligned} u_0 &= 0, \\ u_{k+1} &= 3u_k + 3^k \qquad \text{for } k \geq 0. \end{aligned}$$

Prove that $u_n = n3^{n-1}$ for all non-negative integers n. [4]

4.3 If $f(n) = 3^{2n} + 7$, where n is a natural number, show that $f(n+1) - f(n)$ is divisible by 8. Hence prove by induction that $3^{2n} + 7$ is divisible by 8. [6]

QUESTION 5

5.1 (a) What is a partition of a set S? [2]

(b) Let S be a set and let $\mathscr{R}$ be an equivalence relation on S. Prove that the equivalence classes of $\mathscr{R}$ form a partition of S. [10]

5.2 Define a totally ordered set. [2]

5.3 State whether each of the following subsets U of $\mathbb{N}$ is totally ordered or not if the relation on U is "x divides y":

(a) $U = \{24, 2, 6\}$; [2]

(b) $U = \{3, 5, 15\}$; [2]

(c) $U = \{15, 5, 30\}$; [2]

QUESTION 6

6.1 (a) Define the terms maximal element and minimal element of a poset A with a partial order $\mathscr{R}$' [3]

(b) Let the set $B = \{2, 3, 4, 5, 6, 8, 9, 10\}$ be ordered by the relation $\mathscr{R}$ defined by "x is a multiple of y".

i. Show that $\mathscr{R}$ is an order on B. [6]

ii. Find all maximal elements and all minimal elements of B. [4]

iii. Does B have a first and a last element? Support your answer. [2]

6.2 Prove, by contradiction, that if $A \cap B \subseteq C$ and $x \in B$, then $x \notin (A - C)$. [5]

QUESTION 7

7.1 Using truth tables, prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$. Draw a Venn diagram to illustrate the proof. [10]

7.2 Prove that if $S \subseteq T$ and $T \subseteq \mathbb{R}$, where $S \neq \emptyset$ and $T \neq \emptyset$, and if u is an upper bound for T, then u is an upper bound for S. [3]

7.3 Let $S = \{x \in \mathbb{Q} : x^2 < 2\}$. Prove that $\mathit{inf}\, S = -\sqrt{2}$ and $\mathit{sup}\, S = \sqrt{2}$. [7]

END OF EXAMINATION