UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008/9

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	FOUNDATIONS OF MATHEMATICS
COURSE NUMBER	:	M231
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. State the difference between deductive reasoning and inductive reasoning. Which of the two is a valid form of argument? Explain. [4]

2. Prove that if n is an integer and n^2 is divisible by 2, then so is n. [6]

3. Using the result in part 2, or otherwise, prove that if r is a real number such that $r^2 = 2$, then r is irrational. [10]

QUESTION 2

1. Determine the following sets:

   (a) $\{n \in \mathbb{N} : \exists m \in \mathbb{N} \text{ with } m \leq n\}$; [3]

   (b) $\{n \in \mathbb{N} : \forall m \in \mathbb{N} \text{ we have } m \leq n\}$. [2]

2. Let a be an algebraic number and r a rational number. Show that ra is an algebraic number. [5]

3. Suppose you want to show that $A \Rightarrow B$ is **false**. How should you do this? What should you try to show about the truth of A and B? [2]

4. Apply your answer of part (a) to show that the statement "If x is a real number that satisfies $-3x^2 + 2x + 8 = 0$, then $x > 0$" is false. [3]

5. Write the negation of the statement: "The real-valued function f of one variable is *continuous at the point* x if and only if for every real number $\varepsilon > 0$, there is a real number $\delta > 0$ such that, for all real numbers y with $|x - y| < \delta$, $|f(x) - f(y)| < \varepsilon$." [5]

QUESTION 3

1. Show that $(A \Rightarrow B) \Leftrightarrow (\neg B \Rightarrow \neg A)$ is a tautology. [8]

2. Use truth table analysis to show that:

   (a) $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$; [6]

   (b) $\neg(P \Rightarrow Q) \equiv P \wedge \neg Q$. [6]

QUESTION 4

1. Prove that in any set of $n+1$ pairwise distinct integers, there must be two whose difference is divisible by n. [7]

2. Prove, by the contrapositive method, that if no angle of a quadrilateral $RSTU$ is obtuse, then the quadrilateral $RSTU$ is a rectangle. [6]

3. (a) Show that if r is a nonzero rational number, then $r\sqrt{7}$ is an irrational number. [4]

   (b) Using the result in part (a), or otherwise, show that $\sqrt{28}$ is irrational. [3]

QUESTION 5

1. Prove that $A \subseteq B \Leftrightarrow A \cap B = A$. [5]

2. Using the fact that an implication is equivalent to its contrapositive, prove that, for subsets of a universal set $\mathscr{U}$, $A \subseteq B \Leftrightarrow B^c \subseteq A^c$. [5]

3. Let p_1 and p_2 be distinct prime numbers. Prove that the real numbers $\sqrt{p_1}+\sqrt{p_2}$ and $\sqrt{p_1} - \sqrt{p_2}$ are irrational. [10]

QUESTION 6

1. Using truth tables, analyze the following argument and state whether it is valid or invalid.

   "All Germans are Europeans.
   My neighbor is a European.
   Therefore my neighbor is a German."

   [6]

2. (a) Define an equivalence relation. [2]

   (b) Let $\mathbb{Z}$ be the set of all integers and let

   $$\mathscr{R} = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} : x \equiv y \pmod 3\}$$

   be a relation on $\mathbb{Z}$. Show that $\mathscr{R}$ is an equivalence relation. What are the equivalence classes of $\mathscr{R}$? [12]

QUESTION 7

1. (a) Define a square-free natural number. [2]

   (b) Let b and m be two natural numbers such that b is square-free and m^2 is divisible by b. Prove that m is also divisible by b. [10]

2. Let S and T be finite sets, and let $f : S \longrightarrow T$ be a function. Prove that:

   (a) If f is onto, then $|S| \geq |T|$; [3]

   (b) If f is one-to-one, then $S| \leq |T|$; [3]

   (c) If f is a bijection, then $|S| = |T|$. [2]

END OF EXAMINATION