UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2008/2009

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Given the three points $P(1,-1,2)$, $Q(2,-2,4)$ and $R(2,-1,3)$, find

(i) the angle between $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ [2]

(ii) the area of the triangle whose vertices are given by the three points [2]

(iii) the equation of the plane passing through the three points. [4]

(b) Find the volume of the parallelepiped whose edges are the vectors

$$\mathbf{A} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}, \quad \mathbf{B} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}}, \quad \mathbf{C} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} - 2\hat{\mathbf{k}}.$$

[2]

(c) Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and has half its length. [5]

(d) Find two unit vectors in the $xy-$ plane that are perpendicular to the vector $4\hat{\mathbf{i}} - 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$. [5]

QUESTION 2

(a) In spherical coordinates (ρ, ϕ, θ), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r} = \rho \sin\phi \cos\theta \hat{\mathbf{i}} + \rho \sin\phi \sin\theta \hat{\mathbf{j}} + \rho \cos\phi \hat{\mathbf{k}}.$$

Find the velocity of any particle moving in this coordinate system. [8]

(b) Find a unit vector that is normal to the surface $2x^2 + 4yz - 5z^2 = -10$ at the point $(3, -1, 2)$. [4]

(c) If $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ and $r = |\mathbf{r}|$, show that:

(i) $\nabla r = \dfrac{\mathbf{r}}{r}$,

(ii) $\nabla^2(\log r) = \dfrac{1}{r^2}$. [4,4]

QUESTION 3

If

$$\mathbf{r}(s) = a\cos\left(\frac{s}{\omega}\right)\mathbf{i} + a\sin\left(\frac{s}{\omega}\right)\mathbf{j} + b\frac{s}{\omega}\hat{\mathbf{k}}$$

where s denotes the arc length and a, b and ω are constants, find;

(a) the unit tangent vector $\hat{\mathbf{T}}$ [7 Marks]

(b) the curvature κ [4 Marks]

(c) the unit principal normal $\hat{\mathbf{N}}$ [3 Marks]

(d) the unit binormal vector $\hat{\mathbf{B}}$. [6 Marks]

QUESTION 4

(a) A comet moves in a plane under the gravitational attraction of the sun, which is situated at the origin O. Given that the attractive force between the sun and the comet can be written as

$$f(r) = -\frac{GMm}{r^2};$$

(i) Derive the equations

$$(\ddot{r} - r\dot{\theta}^2) = -\frac{GM}{r^2},$$

$$r^2\dot{\theta} = h,$$ [3,3]

where r and θ are plane polar coordinates, h is a constant, G is the gravitational constant, M is the mass of the sun, and m is the mass of the comet.

(ii) Suppose that at the initial instant, $\theta = 0$, the comet is at distance d from the sun and is moving with speed v in a direction perpendicular to the radius vector from the sun. Show, by means of the substitution $r = \frac{1}{u}$, that the equation of motion of the particle is

$$\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = \frac{GM}{d^2v^2}.$$

[7]

(b) Under the influence of a central force field, a particle moves in a circular orbit through the origin. Find the law of force. [7]

QUESTION 5

(a) A particle of mass m is thrown vertically upwards with initial speed V, and the air resistance at speed v is $m\kappa v^2$, where κ is a constant. Suppose that the displacement of the particle is defined as x and is measured upwards from the ground level.

(i) Show that the height x is given by

$$x = \frac{1}{2\kappa} \ln \left(\frac{g + \kappa V^2}{g + \kappa v^2} \right)$$

[7]

(ii) Show that H, the maximum height reached, is given by

$$H = \frac{1}{2\kappa} \ln \left(\frac{g + \kappa V^2}{g} \right)$$

[3]

(b) A particle is projected vertically upwards with initial speed u. Gravity acts, as does air resistance, which is given by kv per unit mass, where k is a constant and v is the speed of the particle. Find the time taken to reach the maximum height. [10]

QUESTION 6

(a) An inductor of 2 henries, a resistor of 4 ohms, and a capacitor of 0.05 farads are connected in series with a battery of $E = 100$ volts. At $t \leq 0$ the charge on the capacitor and the current in the circuit are zero. Find the charge and current at any time $t > 0$. [8]

(b) Solve the problem in (a) if now the battery is of e.m.f. $E = 100\sin(4t)$. [12]

QUESTION 7

(a) A particle moves on the x axis, attracted to the origin O by a force proportional to its distance from O. If the particle starts from rest at $x = 5$ cm and reaches $x = 2.5$ cm for the first time after 2 seconds, find:

(i) the position at any time t after it starts;

(ii) the magnitude of the velocity at $x = 0$;

(iii) the amplitude, period, and frequency of the vibration; and

(iv) the acceleration. [6,3,2,1]

(b) (i) A 7 kg weight suspended at the end of a vertical spring stretches it 5 cm. Assuming that a damping force numerically equal to 0.2 times the velocity is acting on the system, find the position of the weight at any time t if initially the weight is pulled down 10 cm and released.

(ii) Is the motion in (i) oscillatory, overdamped, or critically damped? [7,1]

END OF EXAMINATION