UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2008/2009

BSc. / BEd. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M 255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) In cylindrical coordinates (r, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{R} = r\cos\theta\hat{\mathbf{i}} + r\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

Show that, in this coordinate system;

(i) the velocity is given by

$$\underline{\mathbf{v}} = \frac{d\mathbf{R}}{dt} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta} + \dot{z}\hat{\mathbf{k}}$$

[6 marks]

(ii) the acceleration is given by

$$\underline{\mathbf{a}} = \frac{d\mathbf{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta} + \ddot{z}\hat{\mathbf{k}}$$

[4 marks]

(b) Find the equation of the plane that contains the point $(2, 1, 0)$ and has a normal vector $\mathbf{n} = (1, 2, 3)$. [6 marks]

(c) For what values of a are $A = a\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $B = 2a\mathbf{i} + a\mathbf{j} - 4\mathbf{k}$ perpendicular? [4 marks]

QUESTION 2

(a) The force acting on a particle of mass m is given in terms of time t by

$$\mathbf{F} = a\cos\omega t\mathbf{i} + b\sin\omega t\mathbf{j}$$

If the particle is initially at rest at the origin, prove that the position at any later time is

$$\underline{\mathbf{r}} = \frac{a}{m\omega^2}(1-\cos\omega t)\mathbf{i} + \frac{b}{m\omega^2}(\omega t - \sin\omega t)\mathbf{j}$$

[6 marks]

(b) A lift ascends 400 metres in 2 minutes traveling from rest to rest. For the first 30 seconds it travels with uniform acceleration, for the last 20 seconds with uniform retardation and for the rest of the time it travels with uniform speed. Calculate

(i) the uniform speed in metres per second; [4 marks]

(ii) the uniform acceleration in metres per second squared [5 marks]

(iii) the time taken by the lift to ascend the first 200 metres. [5 marks]

QUESTION 4

((a) Suppose that a point A has position vector $\mathbf{a}$ and a point B has position vector $\mathbf{b}$. Show that the position vector $\mathbf{r}$ of the point R that divides the line AB in the ratio $\alpha : \beta$ is given by

$$\mathbf{r} = \frac{\beta\mathbf{a} + \alpha\mathbf{b}}{\alpha + \beta}.$$

Hence, deduce the midpoint formula. [7 marks]

(b) If $\mathbf{r}(t) = \mathbf{a}\cos(\omega t) + \mathbf{b}\sin(\omega t)$, where $\mathbf{a}$ and $\mathbf{b}$ are constant non-collinear vectors and ω is a constant scalar, prove that

(i) $\mathbf{r} \times \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \omega(\mathbf{a} \times \mathbf{b})$

(ii) $\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} + \omega^2\mathbf{r} = \mathbf{0}$. [8 marks]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi\mathbf{A})$$

[5 marks]

QUESTION 5

If

$$\mathbf{r}(t) = 4\sin t\hat{\mathbf{i}} + 4\cos t\hat{\mathbf{j}} + 8\hat{\mathbf{k}},$$

find;

(a) the unit tangent vector $\hat{\mathbf{T}}$ [7 Marks]

(b) the curvature κ [4 Marks]

(c) the unit principal normal $\hat{\mathbf{N}}$ [3 Marks]

(d) the unit binormal vector $\hat{\mathbf{B}}$. [6 Marks]

QUESTION 6

(a) The position of a particle moving along the x axis is determined by the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 8x = 20\cos(2t).$$

If the particle starts from rest at $x = 0$, find

(i) x as a function of t,

(ii) the amplitude, period, and frequency after a long time. [7,3]

(b) The weight on a vibrating spring undergoes forced vibrations according to the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4x = 8\sin(\omega t),$$

where x is the displacement from the equilibrium position and ω is a constant. If $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 0$ when $t = 0$, find:

(i) x as a function of t,

(ii) the period of the external force for which resonance occurs. [7,3]

QUESTION 7

(a) From a point O, at height h above sea level, a particle is projected under gravity with a velocity of magnitude $\frac{3}{2}\sqrt{gh}$. Find the two possible angles of projection if the particle strikes the sea at horizontal distance $3h$ from 0. [10 marks]

(b) Two points A and B are at distance d apart. A particle starts from A and moves in the direction $\overrightarrow{AB}$ with initial velocity u and uniform acceleration a. A second particle starts at the same time from B and moves in the direction $\overrightarrow{BA}$ with initial velocity $2u$ and retardation a.

(i) Prove that the particles collide at time $\frac{d}{3u}$ from the beginning of the motion. [5 marks]

(ii) Prove that if the particles collide before the second particle returns to B, then

$$ad < 12u^2.$$

[5 marks]