UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2008/09

BSc.VI

TITLE OF PAPER	:	FLUID MECHANICS
COURSE NUMBER	:	M455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE (5) QUESTIONS 3. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r,\theta,z)$ is cylindrical polar coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\underline{\hat{r}} + \frac{1}{r}\frac{\partial\psi}{\partial r}\underline{\hat{\theta}} + \frac{\partial\psi}{\partial r}\underline{\hat{k}}.$$

The divergence and curl of a vector field

$$\underline{v} = v_r\underline{\hat{r}} + v_\theta\underline{\hat{\theta}} + v_z\underline{\hat{k}}$$

in cylindrical polar coordinates are

$$\nabla\cdot\underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\},$$

and

$$\nabla\times\underline{v} = \frac{1}{r}det\begin{bmatrix} \underline{\hat{r}} & r\underline{\hat{\theta}} & \underline{\hat{k}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{bmatrix}.$$

The divergence of a vector

$$\underline{v} = v_r\underline{\hat{r}} + v_\lambda\underline{\hat{\lambda}} + v_\theta\underline{\hat{\theta}}.$$

is spherical coordinates

$$\nabla\cdot\underline{v} = \frac{1}{r^2}\frac{\partial(r^2v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta v_\theta)}{\partial\theta}.$$

The convective derivative, Laplacian and strain and shear stress in cylindrical coordinates are:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial_r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}.$$

$$e_{r\theta} = \frac{1}{2}r\frac{\partial}{\partial r}(\frac{V_0}{r}) + \frac{1}{2}\frac{1}{r}\frac{\partial V_r}{\partial\theta}, \quad s_{r\theta} = 2\mu e_{r\theta}$$

Identities:

$$\underline{V} \cdot \nabla \underline{V} = \nabla(\frac{1}{2}V^2) - \underline{V} \times \underline{\omega}.$$

$$\nabla \times (\nabla \times \overline{A}) = \nabla\nabla \cdot \overline{A} - \nabla^2 \overline{A}.$$

QUESTION 1

(a) Consider a flow field with tangential motion $V_r = 0$ and $V_\theta = f(r)$.

(i) For rigid-body rotation $V_\theta = Ar$ find the

- rotation,

-vorticity,

-circulation.

(ii) Show that it is possible to choose $f(r)$ so that the flow is irrotational, and find the circulation for that flow.

(b) (i) Derive the continuity, or mass conservation equation for a fluid.

(ii) Write down the equation in (b)(i) for a steady flow.

(c) For a steady incompressible two-dimensional flow in the xy-plane the x component of velocity is $U = ax$, where a is a constant

(i) determine a possible y component.

(ii) How many possible y components are there? [6,4,5,1,3,1]

QUESTION 2

(a) A tank partly filled with water (to a depth d) is subject to constant linear horizontal acceleration a_x. Tank height is H; length parallel to direction of motion is b; width perpendicular to direction of motion is c. Find

(i) shape of the free surface,

(ii) the allowable water depth, d, to avoid spilling.

(b) Describe two methods of modelling continuous medium.

(c) Describe the continuum model of a fluid.

(d) Derive a formula for the convective derivative of the density. [5,2,4,4,5]

QUESTION 3

(a) Consider a piston-cylinder apparatus. At one instant when the piston is $L_o = 0.15m$ away from the closed end of the cylinder, the gas density is uniform at $\rho = 16kg/m^3$ and the piston begins to move away from the closed end at $V = 12m/sec$. The gas motion is one-dimensional and propotional to the distance from the closed end; it varies linearly from zero at the end to $u = V$ at the pistion.

(i) Evaluate the rate of change of gas density at this instant.

(ii) Obtain an expression for the average density as a function of time.

(b) Let $\overline{V} = mr^{-2}\hat{r}$ in spherical coordinates.

(i) Show that $\overline{V}$ satisfies the continuity equation for incompressible flow, except at the origin 0.

(ii) Let O be lye on a smooth surface S.

What is the volume flow rate through S? [5,6,4,5]

QUESTION 4

(a) A cylindrical container, partly filled with liquid, is rotated at a constant angular velocity ω about its vertical axis. After a short time there is no relative motion, the liquid rotates as a rigid body. Determine the shape of the free surface if the radius of the cylinder is R and the original surface height, in the absence of rotation is h_0.

(b) An infinite plate, is moved over a second plate on a layer of liquid with velocity $V = 0.3m/sec$. For small gap width $d = 0.3mm$ we assume linear velocity profile. The liquid viscosity is $\mu = 0.65 \times 10^{-3}kg/m.s$ and specific gravity $SG = 0.88$. Calculate

(i) the kinematic viscosity,

(ii) the shear stress on the lower plate.

(c) Give the definitions of a vortex line, a vortex surface, a vortex tube and the strength of vortex tube. [10,2,4,4]

QUESTION 5

(a) Air flows steadily and at low speed through a horizontal nozzle discharging to the atmosphere. At the nozzle inlet, the area is $0.1m^2$. At the nozzle exit, the area is $0.02m^2$. The flow is incompressible, and frictional effects are negligible. Determine the pressure required at the nozzle inlet to produce an outlet speed of 50m/s. For air at standard conditions $\rho = 1.23kg/m^3$.

(b) Give the definition of a Newtonian fluid.

(c) Use (b) to find the dimension of

(i) Viscosity,

(ii) Kinematic viscosity.

(d) Derive the Navie-Stokes equation in the form

$$\rho\frac{DV_i}{Dt} = \rho F_i + \frac{\partial}{\partial x_j}\sigma_{ij}, \quad i = 1, 2, 3.$$

[7,2,2,2,7]