
University of Swaziland



Final Examination, December 2009

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Simplify

$$\log_2(2x^2) - 2\log_2(2x) + \log_2\left(\frac{\sqrt{8}}{x}\right) + \frac{1}{2}\log_2\left(\frac{x^2}{32}\right). \quad [6]$$

(b) The 6th term of an AP is 21 and the sum of the first 17 terms is zero. Find the sum of all the terms between the 20th and the 40th inclusive. [6]

(c) Find the quotient and remainder of the division

$$\frac{x^5 - 2x^4 + x^2 - 8}{2x - x^2}. \quad [8]$$

Question 2

(a) Evaluate $(1 - i\sqrt{3})^6$ and express your answer in the form $a + ib$, using the

i. Binomial theorem [5]

ii. de Moivre's theorem. [5]

(b) Find the equation of the circle centred on the line $2x + 5y + 1 = 0$, and passing through the points $(2, 3)$ and $(-1, 6)$. [10]

Question 3

(a) Evaluate

$$\begin{vmatrix} 4 & -3 & 2 & 0 \\ 5 & 1 & 0 & -2 \\ 2 & -2 & 0 & 3 \\ 0 & 4 & -2 & 2 \end{vmatrix}. \quad [6]$$

(b) Solve for x , given

i. $\log_7(x + 1) + \log_7(x - 5) = 1$ [6]

ii. $\cos 2x + \sin x = 0, 0 < x \leq 2\pi.$ [8]

Question 4

(a) State the

i. *Rational Root Theorem*, and the [3]

ii. *Remaider Theorem*. [3]

(b) Use the Rational Root Theorem, the Remaider Theorem and synthetic division to solve

$$6x^4 - 5x^3 - 12x^2 + 5x + 6 = 0. \quad [7]$$

(c) Find the first 4 terms of the binomial expansion of

$$\left(\frac{1}{x^2} - \frac{2y^2}{x}\right)^{\frac{1}{2}}. \quad [7]$$

Question 5

(a) Describe the curve defined by the equation

$$9x^2 - 4y^2 - 36x + 32y + 8 = 0.$$

Make a sketch of the curve. [10]

(b) Use mathematical induction to prove that

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}, \quad n \geq 1. \quad [10]$$

Question 6

(a) Use Cramer's rule to solve

$$3x + 4y - z = -7,$$

$$x - 5y + 2z = 19,$$

$$5x + y - 2z = 5.$$

[15]

(b) Find the middle term of the binomial expansion of

$$\left(\frac{2}{x} - x^2\right)^{20}. \quad [5]$$

Question 7

(a) In a certain city, the number of people exposed to an airborne viral infection t days after an outbreak is given by the function

$$P(t) = 76,000(1 - e^{-0.012t}).$$

i. How many people will be infected after 10 days? [4]

ii. How long (in days and hours) does it take for $\frac{1}{5}$ of the population to be infected? [8]

(b) Prove that

$$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2. \quad [8]$$
