University of Swaziland

Supplementary Examination – July 2010

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.

2. Each question is worth 20%.

3. Answer ANY FIVE questions.

4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Expand
$$\left(a^2b - \frac{2a}{b^2}\right)^5$$
and simplify term by term. [8]

(b) Expand
$$\frac{(1+i)(1+2i)(1+3i)}{3-i}$$
and express your answer in the form $a + ib$. [6]

(c) Use synthetic division to work out
$$\frac{2x^4 - 2x^2 - 3x + 5}{x - 2}.$$
[6]

Question 2

Use Cramer's rule to solve
$$
\begin{aligned}
x - 2y + z &= 7, \\
2x + y - z &= 0, \\
3x + 2y - 2z &= -2.
\end{aligned}
$$

[20]

Question 3

(a) Solve

    i. $\log_5\left(\log_4 x^2\right) = 0$ [4]

    ii. $3^x = 2 \cdot 5^{1-x}$ [8]

iii. $\begin{vmatrix} x & 4 \\ x & x \end{vmatrix} = \begin{vmatrix} -5 & 4 \\ 7 & -8 \end{vmatrix}.$ [8]

Question 4

(a) Find the cube roots of -27. [10]

(b) Prove that

$$\frac{\cos\theta}{1 - \tan\theta} + \frac{\sin\theta}{1 - \cot\theta} = \cos\theta + \sin\theta. \qquad [10]$$

Question 5
Solve

a. $\sin 2x + \cos x = 0, \quad 0 < x \leqslant 2\pi$ [8]

b. $x^4 - 2x^3 + x^2 + 2x - 2 = 0.$ [12]

Question 6

(a) Divide

$$\frac{x^4 - x^3 + x - 3}{x^2 - 1}. \qquad [12]$$

(b) Find the first 4 terms of the binomial expansion of

$$\left(\frac{1}{x} - x^2\right)^{-2}. \qquad [8]$$

Question 7

(a) A pendulum swings such that the length of the arc swept by its bob is always 90% the length of the previous swing. Find the total distance travelled by the

bob in coming to rest if the initial swing has an arc of
2 metres. [5]

(b) Prove that $P(n) = 2^{4n} - 1$, $n \geqslant 1$, is always divisible
by 15. [10]

(c) Find the value of

$$\cos\left(\tan^{-1}\sqrt{3}\right).$$ [5]