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# University of Swaziland



Final Examination, 2009/10

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BSc I, Bass I, BEd I, EEng I

**Title of Paper** : Introduction to Calculus

**Course Number** : M115

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

### Question 1

(a) Find the following limits

(i)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

(ii)  $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$

[6]

(b) Find the integral

$$\int \sin^3 x \cos^3 x dx. \quad [6]$$

(c) Use integration by partial fractions to evaluate

$$\int \frac{2x^2 + x - 1}{x^3 - x^2} dx. \quad [8]$$

### Question 2

(a) Find the area of the region lying above the  $x$ -axis and under the parabola  $y = 4x - x^2$ . [6]

(b) Use integration by parts to evaluate

$$\int x^3 \sin x dx. \quad [7]$$

(c) Determine the value of  $k$  given that the tangent to the curve

$$y = x^2 + kx + 1$$

at the point  $(0, 1)$  is parallel to the straight line

$$y = 2x + 4.$$

[7]

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**Question 3**

(a) Use the limit definition to evaluate the derivative of

$$f(x) = \frac{1}{\sqrt{1+2x}}. \quad [8]$$

(b) Evaluate the following integral

$$\int \frac{\cos x}{1 - \sin x} dx. \quad [6]$$

(c) Use implicit differentiation to find  $\frac{dy}{dx}$

$$e^y - e^x = e^{y-x}. \quad [6]$$

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**Question 4**

(a)

(i) Prove the following reduction formula

$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + (n-1) \int \cos^{n-2} x dx. \quad [5]$$

(ii) Use (i) to evaluate

$$\int \cos^5 x dx. \quad [5]$$

(b) Find  $\frac{dy}{d\theta}$  for

$$\theta \cos y = \sin(\theta + y). \quad [5]$$

(c) Find  $\frac{dy}{dx}$  for

$$y = x^2 \operatorname{arcsec} \left( \frac{2}{x} \right). \quad [5]$$

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### Question 5

(a)

(i) Find the expression for  $\frac{d^n y}{dx^n}$  ( $n$  is a positive integer) for

$$y = \frac{1}{(1+2x)^2}. \quad [5]$$

(ii) Find the first and second derivative of

$$y = x \cos x. \quad [5]$$

(b) Find the derivative of  $y$  with respect to  $x$  for

$$y = x^{2x}. \quad [5]$$

(c) Find the equation of the tangent to the curve

$$y = x^3 - 3x^2 + 7$$

at the point  $(2, 3)$ . [5]

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### Question 6

(a) Evaluate the definite integral

$$\int_0^1 x(x^2 + 3)^8 dx. \quad [6]$$

(b) Use Leibnitz's rule to find  $f^{(4)}(x)$  for

$$f(x) = x^6 \ln x. \quad [8]$$

(c) Find  $\frac{dy}{dx}$  for the following

$$\begin{aligned} y &= 3t - t^3 \\ x &= 2t - t^2. \end{aligned}$$

[3]

(d) Evaluate

$$\int \left( x^4 + 3x^2 + \frac{1}{x} + \frac{1}{x^5} \right) dx. \quad [3]$$

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### Question 7

(a) Use trigonometric substitution to evaluate the following.

(i)  $\int \frac{dx}{x\sqrt{x^2-1}}$

(ii)  $\int \frac{x^2}{(9-x^2)^{\frac{3}{2}}} dx$

[10]

(b) Find the slope of the tangent to the curve  $y = x^4 - 2x^3 + 3$  at the point  $(-1, 6)$ . [3]

(c) Find the area of the region enclosed between  $y = 10 + 3x - x^2$  and  $y = 2x + 4$ . [7]

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