
University of Swaziland



Supplementary Examination, 2009/10

BSc I, Bass I, BEd I, EEng I

Title of Paper : Introduction to Calculus

Course Number : M115

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

- (a) Use partial fractions to evaluate

$$\int \frac{x^4 - x^3 - x - 1}{x^3 - x^2} dx. \quad [8]$$

- (b) Show that

$$\int x\sqrt{1+x} dx = \frac{2}{15}(1+x)^{\frac{3}{2}}(3x-2) + C$$

in two ways:

- (i) Using integrations by parts
(ii) Using the substitution $u = \sqrt{1+x}$.

[12]

Question 2

- (a) Find the equation of the tangent to the curve

$$x^4 - y^4 - 4xy = 7$$

at the point $(2, 1)$. [6]

- (b) Evaluate the indefinite integral

$$\int \frac{\sin x}{1 + \cos^2 x} dx. \quad [6]$$

- (c) Find y' if

(i) $y = x^{2/x}$ [4]

(ii) $y = \tanh(e^{-2x})$ [4]

Question 3

(a)

(i) Evaluate the following limit

$$\lim_{x \rightarrow 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h} \right)$$

(ii) Evaluate $\frac{d^2y}{dx^2}$ for

$$xy + y^2 = 1.$$

[10]

(b) Perform the following integrations.

(i) $\int x\sqrt{1-x} dx$

(ii) $\int \sqrt{\cos x} \sin 2x dx$

[10]

Question 4

(a) Integrate the following

(i) $\int \frac{x^3}{x^2 - 2x + 1} dx$

(ii) $\int \frac{dx}{1 + \cos x}$

[10]

(b) Use the limit definition to find the derivative of

$$y = \sqrt{1 + 2x}. \quad [5]$$

(c) Evaluate the following limit

$$\lim_{x \rightarrow 0} \left(\frac{4 - x^2}{3 - \sqrt{x^2 + 5}} \right). \quad [5]$$

Question 5

(a) Integrate

(i) $\int \sqrt{3 - 2x - x^2} dx \quad [5]$

(ii) $\int \sin^2 x dx \quad [5]$

(b) Derive the reduction formula

$$\int (\ln x)^n dx = x (\ln x)^n - n \int (\ln x)^{n-1} dx. \quad [5]$$

(c) Use (b) above to evaluate

$$\int (\ln x)^2 dx. \quad [5]$$

Question 6

(a) Find $\frac{dy}{dx}$ for the following

$$y = u^2 - 1; \quad x = u^2 + 1. \quad [5]$$

(b) Find $\frac{dy}{dx}$ implicitly for

$$x^2 y^2 = x^2 + y^2. \quad [5]$$

(c) Integrate

$$\int \left(\frac{2 - 5x^3 - 7x^5}{\sqrt{x}} \right) dx. \quad [5]$$

(d) Find y' for

$$y = x^{2x}. \quad [5]$$

Question 7

(a) Find the area bounded on the right by $x + y = 2$, on the left by $y = x^2$ and below by the x -axis. [7]

(b) Derive the reduction formula

$$\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx. \quad [4]$$

(c) Use (b) to evaluate

$$\int \cos^3 x dx. \quad [4]$$

(d) Show that the function

$$y = a \sin kx + b \cos kx,$$

where a , b and k are constants, is a solution of the equation

$$y'' + k^2 y = 0. \quad [5]$$
