UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2009/2010

BSc. /BEd. /BEng. /B.A.S.S II

TITLE OF PAPER	:	CALCULUS I
COURSE NUMBER	:	M 211
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) State Rolle's theorem. [2 marks]

   (b) For what values of m and c does the function

$$f(x) := \begin{cases} mx - 1, & \text{if } 0 \leq x < 1 \\ x^2 + 2x + c, & \text{if } 1 \leq x \leq 2 \end{cases}$$

   satisfy the hypothesis of Rolle's theorem? [5 marks]

   (c) Sketch the graph of the function

$$f(x) = \frac{e^x}{1 + e^x}$$

   Indicate all intercepts, extrema, points of inflection and asymptotes where necessary. [13 marks]

QUESTION 2

2. (a) Evaluate the following limits clearly showing your steps.

   (i) $\lim_{x \to 0}(x \cot x)$ [6 marks]

   (ii) $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x$ [7 marks]

   (b) State the Mean Value theorem. [4 marks]

   (c) Use the Mean Value theorem to show that

$$|\sin x - \sin y| \leq |x - y|$$

   for any real numbers x and y. [3 marks]

QUESTION 3

3. (a) Consider the function

$$f(x) = xe^{-x} \text{ on the closed interval } [-1, 1]$$

   (i) Identify local extreme values of f saying where they are taken on. [5 marks]

   (ii) Determine global extreme values of f? [5 marks]

(b) Find the area of the region in the first quadrant that is bounded below by the curve $y = x^2$ and bounded above by the lines $y = 2x$ and $y = 1$. Include a sketch of the region in your answer. [10 marks]

QUESTION 4

4. (a) The region bounded by $x + y = 5$ and $xy = 4$ is revolved about the y-axis to generate a solid. Use the washer method to find the volume of the solid. [10 marks]

(b) The region in the first quadrant bounded by $y = 4 - x^2$ and the x and y axes is revolved about the x-axis to generate a solid. Use the disk method to find the volume of the solid. [10 marks]

QUESTION 5

5. (a) Find the arc length of the curve

$$y = x^{\frac{3}{2}} \text{ from } x = 0 \text{ to } x = 4$$

[10 marks]

(b) Determine the arc length of the curve that is represented parametrically by

$$x = t - \sin t,\ y = 1 - \cos t,\ \text{for } 0 \le t \le \frac{\pi}{2}$$

[10 marks]

QUESTION 6

6. (a) Find the limit of the sequence

$$a_n = \left(\frac{1}{n}\right)^{\frac{1}{\ln n}}$$

[6 marks]

(b) (i) State the Non-decreasing Sequence theorem. [2 marks]

(ii) Consider the sequence $\{a_n\}$ with

$$a_n := \frac{n}{n+1}$$

A. Show that $\{a_n\}$ is non-decreasing. [4 marks]

B. Prove that $\{a_n\}$ is bounded from above. [4 marks]

C. Deduce that $\{a_n\}$ is convergent clearly stating any theorem used. [2 marks]

D. Hence evaluate $\lim_{n\to\infty} a_n$. [2 marks]

QUESTION 7

7. For each of the following series, use any appropriate test to check for convergence or divergence. State any test used.

(a) $\sum_{n=1}^{\infty} \frac{2^n}{n+1}$ [6 marks]

(b) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$ [8 marks]

(c) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$ [6 marks]