UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2009/2010

BSc. /BEd. /BEng. /B.A.S.S II

TITLE OF PAPER : CALCULUS I

COURSE NUMBER : M 211

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :
1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) State the Mean Value theorem. [2 marks]

   (b) For what values of a and b does the function

$$f(x) := \begin{cases} ax + b, & \text{if } x < 0 \\ e^x, & \text{if } x \geq 0 \end{cases}$$

   satisfy the hypothesis of the Mean Value theorem? [5 marks]

   (c) Sketch the graph of the function

$$f(x) = x^4 + 2x^3$$

   Indicate all intercepts, extrema, points of inflection and asymptotes where possible. [13 marks]

QUESTION 2

2. (a) Evaluate the following limits clearly showing your steps.

   (i) $\lim_{x \to 0+} (x \ln x)$ [6 marks]

   (ii) $\lim_{x \to 0} (x + e^x)^{\frac{1}{x}}$ [7 marks]

   (b) Show that the function $f(x) = (x - 1)e^x$ is increasing for $x > 0$. [4 marks]

   (c) Suppose that f is differentiable on an interval I and suppose that $-2 \leq f'(x) \leq 2$ for all points x in I. Then, use the Mean Value theorem to show that

$$|f(x_1) - f(x_2)| \leq 2|x_1 - x_2|$$

   whenever both x_1 and x_2 are in I. [3 marks]

QUESTION 3

3. (a) Consider the function

$$f(x) = \frac{1}{x} \text{ on the closed interval } [-2, 1]$$

(i) Identify local extreme values of f stating where they occur. [7 marks]

(ii) Determine the global extreme values of f. [3 marks]

(b) Find the area of the region in the first quadrant that is bounded by the x-axis and the curves $y = x^2$ and $y = 1 - x^2$.
Include a sketch of the region in your answer. [10 marks]

QUESTION 4

4. (a) The region bounded by $x + y = 5$ and $xy = 4$ is revolved about the x-axis to generate a solid. Find the volume of the solid. [10 marks]

(b) The region in the first quadrant bounded by $y = 4 - x^2$ and the x and y axes is revolved about the y-axis to generate a solid. Find the volume of the solid. [10 marks]

QUESTION 5

5. (a) Determine the arc length of the curve that is represented parametrically by

$$x = 1 - \cos t,\ y = t - \sin t,\ \text{for } 0 \leq t \leq \frac{\pi}{2}$$

[10 marks]

(b) Find the arc length of the curve

$$y^2 = x^3 \text{ from } (x, y) = (0, 0) \text{ to } (1, 1)$$

[10 marks]

QUESTION 6

6. (a) Find the limit of the sequence

$$a_n = \left(\frac{n}{n+1}\right)^n$$

[6 marks]

(b) (i) State the Non-increasing Sequence theorem. [2 marks]

(ii) Consider the sequence $\{a_n\}$ with

$$a_n := \frac{n+1}{n}$$

A. Show that $\{a_n\}$ is non-increasing. [4 marks]

B. Prove that $\{a_n\}$ is bounded from below. [4 marks]

C. Deduce that $\{a_n\}$ is convergent clearly stating any theorem used. [2 marks]

D. Hence evaluate $\lim_{n\to\infty} a_n$. [2 marks]

QUESTION 7

7. For each of the following series, use any appropriate test to check for convergence or divergence. State any test used.

(a) $\sum_{n=1}^{\infty} \frac{2}{10^n}$ [6 marks]

(b) $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$ [8 marks]

(c) $\sum_{n=1}^{\infty} \frac{n!}{e^n}$ [6 marks]