University of Swaziland

Final Examination, 2009/2010

BSc II, Bass II, BEd II

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

(a) Consider the curve with polar equation $r = \tan\theta \sec\theta$. First find its Cartesian equation and then describe or identify the curve. [5]

(b) Find the Cartesian coordinates of the point with polar coordinates $\left(2, -\frac{7\pi}{6}\right)$. [3]

(c) Find the polar coordinates (r, θ), with $r > 0$, $0 \leq \theta < 2\pi$, of the point with Cartesian coordinates $(1, -\sqrt{3})$. [4]

(d) Find the area of the region under the curve $r = \sin\theta$ in the sector $\pi/3 \leq \theta \leq 2\pi/3$. [8]

QUESTION 2

(a) Suppose z is defined implicitly as a function of x and y by the equation

$$x^2 + y^2 + z^2 = 3xyz.$$

Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [10]

(b) Let $z = e^r \cos\theta$, $r = st$, $\theta = \sqrt{s^2 + t^2}$. Use the Chain Rule to find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$. [10]

QUESTION 3

(a) A circle of radius a centred at $(a, 0)$ has Cartesian equation

$$(x - a)^2 + y^2 = a^2.$$

By first converting the equation to its polar equivalent and using the arc length formula, show that the circle has circumference $2\pi a$. [10]

(b) Use the definition of the limit to prove that

$$\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2 + y^2} = 0.$$

[10]

QUESTION 4

(a) Evaluate

$$\int_0^1 \int_x^1 \sin(y^2)\, dy dx$$

by first changing the order of integration. [10]

(b) Find the volume of the solid that lies under the plane $3x + 2y + z = 12$ and above the rectangle $R = \{(x, y) | 0 \leq x \leq 1, -2 \leq y \leq 3\}$. [10]

QUESTION 5

(a) Find the following limit, if it exists, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{y^4}{x^4+3y^4}.$$

[5]

(b) Define continuity of a function $f(x,y)$ at a point (x_0, y_0). [5]

(c) Determine the set of points at which the function

$$f(x,y) = \frac{\sin(xy)}{e^x - y^2}$$

is continuous. [5]

(d) Use polar coordinates to evaluate

$$\lim_{(x,y)\to(0,0)} \frac{x^3+y^3}{x^2+y^2}.$$

[5]

QUESTION 6

(a) Use polar coordinates to evaluate

$$\iint_D e^{-x^2-y^2}\, dA$$

where D is the region bounded by the semicircle $x = \sqrt{4-y^2}$ and the y-axis. [10]

(b) Find the area of the part of the surface of the paraboloid $z = x^2 + y^2$ that lies under the plane $z = 9$. [10]

QUESTION 7

(a) Evaluate $\iiint_E 2x\, dV$ where $E = \left\{(x,y,z) | 0 \le y \le 2, 0 \le x \le \sqrt{4-y^2}, 0 \le z \le y\right\}$. [10]

(b) Use spherical coordinates to evaluate $\iiint_E z\, dV$ where E lies between the spheres $x^2+y^2+z^2 = 1$ and $x^2+y^2+z^2 = 4$ in the first octant. [10]

END OF EXAMINATION PAPER