UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2009/10

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) For which values of t are $tx - 2y + 8 = 0$ and $3tx + 6y + 4 = 0$ the equations of

(i) parallel lines

(ii) perpendicular lines?

[2,2]

(b) Describe the solution set of

$$x^2 + y^2 - x - y + 1 = 0$$

[3]

c) If θ is the angle between the vectors $\overline{a} = (1, 1, 1)$ and $\overline{b} = (2, 3, 2)$. Find

(i) $\cos\theta$ using a scalar product,

(ii) $\sin\theta$ using a vector product.

[5,5]

(d) What is a geometrical meaning of a triple product

$$\overline{a} \cdot (\overline{b} \times \overline{c})?$$

Explain.

[3]

QUESTION 2

(a) Consider a square matrix. Give the definitions of

(i) Minor, (ii) Cofactor, (iii) Determinant.

[2,2,2]

(b) Find the inverse and check the result or state that inverse does not exist, giving the reason

(i) $\begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$, (ii) $\begin{bmatrix} 1 & -2 & 0 \\ 3 & 1 & 5 \\ -1 & 2 & 3 \end{bmatrix}$.

[2,6]

(c) Solve the following system by Gauss elimination method

$$\begin{aligned} -x_1 + x_2 + 2x_3 &= 2 \\ 3x_1 - x_2 + x_3 &= 6 \\ -x_1 + 3x_2 + 4x_3 &= 4. \end{aligned}$$

[6]

QUESTION 3

(a) Water is running out of a conical funnel at rate $2cm^3/sec$. Radius at top and height of funnel are 6cm and 10cm, respectively. How fast is the water level dropping when the water is 4cm deep?

[7]

(b) (i) State, and

(ii) prove Rolle's theorem

[1,4]

(c) Apply the L'Hospital rule to evaluate the following limits

(i) $\lim_{x\to 0^+} \frac{e^x - 1}{x^2}$.

(ii) $\lim_{x\to 0}(\cot x - \frac{1}{x})$.

[3,5]

QUESTION 4

(a) Use the Taylor series expansion to state and prove

(i) necessary, and

(ii) sufficient conditions theorem for $f(x)$ to have a minimum at x^*.

[4,4]

(b) (i) Use the quadratic approximation formula to compute $ln(1+x)$ for small $|x|$, and estimate the error.

(ii) Use the results from (i) to calculate ln 1.2.

[6,3]

(c) Find four terms of the Maclaurin's series for $\sin x$.

[3]

QUESTION 5

(a) Find the partial derivatives (at $x = 2, \quad y = 3$) of

$$f(x,y) = 3x^3y + 4xy^2 - 2x + 4y - 8.$$

[5]

(b) Find the partial derivatives with respect to u and v of $f(x,y) = exp(xy)$, where $x = u^2$ and $y = uv$.

[7]

(c) Find and classify all stationary points of the following functions

(i) $x^2 + 2y^2$,

(ii) $x^2 - 2y^2$.

[4,4]

QUESTION 6

(a) Apply Lagrange's method to find the minimum and the maximum of $x_1 + x_2$, provided

$$x_1^2 + x_2^2 = 1.$$

[8]

(b) Find the area of the region enclosed between the curves $y = x^3$, $y = 2x$, $x = 0$ and $x = 1$.

~~[6]~~ 4

(c) Find the volume of a right cone of radius a and height h by using

(i) $V = \int_a^b A(x)dx$,

Where $A(x)$ is an area of the cross-section x,

(ii) formula for the volume of the solid of revolution

~~[6,6]~~ [4,4]

QUESTION 7

(a) Find the area of surface generated by rotating $z = x^2$ from $x = 0$ to $x = \sqrt{2}$ about z-axis

[6]

(b) Compute the volume of solid under surface $z = 4 - x - y$ and over a region R, where

$R = \{(x, y) : y \leq 4 - x, y \geq 0, x \geq 0\}$.

[7]

(c) Pass to polar coordinates to evaluate

$$\iint_D exp(x^2 + y^2)dxdy,$$

$D = \{(x, y) : 1 \leq x^2 + y^2 \leq 3, x \geq 0, y \geq 0\}$.

[7]