UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2009/10

BSc./B.Ed./B.A.S.S II

TITLE OF PAPER	:	LINEAR ALGEBRA
COURSE NUMBER	:	M220
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Give the definition of each of the following

i A vector space

ii An orthogonal matrix

iii A symmetrix matrix

iv A skew-symmetric matrix

[10]

(b) Find the eigenvalues and the corresponding eigenvectors for

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & -1 \\ 4 & -4 & 5 \end{pmatrix}$$

[10]

QUESTION 2

(a) Determine whether the following mappings are linear transformations

i $T : R^3 \to R^2$ defined by $T(x, y, z) = (x + y - z, 2x + y)$

ii $T : R^3 \to R^3$ defined by $T(x, y, z) = (x + 1, y)$

[10]

(b) Prove that the set $B = \{x^7 + 1, x - 1, 2x + 2\}$ is a basis for the vector space $V = P_2(x)$

[10]

QUESTION 3

(a) Let $T : R^3 \rightarrow R^2$ be $T(x, y, z) = (x + y + z, x + 2y + 3z)$

i Find the standard matrix of T.

ii Find the matrix of T relative to the R-bases

$$B_1 = \{(1,1,0),(0,1,1),(0,0,1)\} \quad B_2 = \{(1,2),(1,3)\}$$

[10]

(b) Verifty the Cayley-Hamilton theorem for

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 5 \\ 3 & 2 & 1 \end{pmatrix}$$

[10]

QUESTION 4

(a) Find the inverse of the matrix A in two ways

i using the augmented matrix $[A : I]$

ii by computing a product $E_k E_{k-1} \cdots E_2 E_1$ of elementary matrices

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -2 \end{pmatrix}$$

[12]

(b) Prove that if A and B are both non -singlar $n \times n$ matrices, then the product AB is also non singular and $(AB)^{-1} = B^{-1}A^{-1}$

[4]

(c) Prove that $[M_i(\alpha)]^{-1} = M_i\left(\frac{1}{\alpha}\right) \alpha \neq 0$

[4]

QUESTION 5

(a) Solve the system

$$\begin{aligned} 2x_1 + 5x_2 - 8x_3 + 6x_4 &= 4 \\ x_1 + 2x_2 - 3x_3 + 4x_4 &= 1 \\ x_1 + 4x_2 + 7x_3 + 2x_4 &= 8 \end{aligned}$$

[8]

(b) For which k does the following system have non-trivial solutions

$$\begin{aligned} kx_1 + 2x_2 - x_3 &= 0 \\ (k+1)x_1 + kx_2 - ox_3 &= 0 \\ -x_1 + kx_2 + kx_3 &= 0 \end{aligned}$$

[8]

(c) Determine whether the vectors are linearly independent

$(2, 4, 0, 4, 3), (1, 2, -1, 3, 1), (-1, -2, 5, -7, 1)$

[4]

QUESTION 6

(a)

i Give the definition of a basis of a vector space

ii Determine whether the vectors $(1,1,1),(1,2,3)$ and $(2,-1,1)$ form a basis for R^3

[8]

(b) Use the adjoint of A to find A^{-1} where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

[7]

(c) Show that $\begin{pmatrix} 0 & 0 & 5 \\ 0 & 0 & -1 \\ -5 & 1 & 0 \end{pmatrix}$ is skew symmetric

[5]

QUESTION 7

(a) Let V be a vector, S_1 and S_2 be finite sets of non-zero vector in V such that $S_1 \subset S_2$ Show that

i S_1 linearly dependent $\Rightarrow S_2$ is also linearly dependent

ii S_2 linearly independent $\Rightarrow S_1$ is also linearly independent

[10]

(b) Prove that if a homogeneous system has more unknowns than the number of equations then it always has a non-trivial solution.

[10]