UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2009/2010

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector of a moving particle is given by

$$\mathbf{r} = 3\cos(2t)\hat{\mathbf{i}} + 3\sin(2t)\hat{\mathbf{j}} + (8t - 4)\hat{\mathbf{k}}.$$

Find

(a) the velocity [1]

(b) the speed [1]

(c) the acceleration [1]

(d) the magnitude of the acceleration [1]

(e) the unit tangent vector [2]

(f) the curvature [5]

(g) the radius of curvature [1]

(h) the unit principal normal [2]

(i) the normal component of acceleration [2]

(j) the unit binormal vector. [4]

QUESTION 2

(a) A train takes time T to perform a journey. It travels for time $\frac{T}{n}$ with uniform acceleration, then for time $(n-2)\frac{T}{n}$ with uniform speed V, and finally for time $\frac{T}{n}$ with constant retardation. Prove that its average speed is

$$(n-1)\frac{V}{n}.$$

If the length of this journey is 64 km, the time taken on the whole journey is 60 minutes, and the uniform speed is 96 km/h, find the time which is occupied in traveling with the uniform speed. [6]

(b) Particle A, initially at rest, is projected from the origin with acceleration $\frac{\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}$. Particle B, at rest at the point $(\sqrt{3}, 0)$, is projected at the same instant with acceleration $\frac{1}{2}\hat{\mathbf{j}}$. Show that the particles collide and that the time of collision is $t = 2$. [7]

(c) A particle moving in a straight line is acted upon by a retarding force of kv^3 per unit mass, where k is a constant and v is the speed. Show that after traveling a distance x, the speed and time taken are given by

$$v = \frac{u}{1 + kux} \quad \text{and} \quad t = \frac{1}{2}kx^2 + \frac{x}{u},$$

where u is the initial speed. [7]

QUESTION 3

(a) If $\mathbf{a} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{b} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$, prove that $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. [2]

(b) A car with initial speed u accelerates uniformly over a distance of $2s$ which it covers in time t_1. It is then stopped by being retarded uniformly to rest over a distance s, which it covers in time t_2. Prove that

$$\frac{u}{2s} = \frac{2}{t_1} - \frac{1}{t_2}.$$

[3]

(c) A particle of unit mass is thrown vertically upwards with initial speed V. The air resistance at speed v is kv^2 per unit mass, where k is a constant.

(i) Show that H, the maximum height reached, is given by

$$H = \frac{1}{2k}\ln\left(\frac{g + kV^2}{g}\right),$$

and that the time T taken to reach this height is

$$T = \frac{1}{\sqrt{gk}}\tan^{-1}\left[\left(\frac{k}{g}\right)^{\frac{1}{2}}V\right].$$

[9]

(ii) Show that the particle return to the point of projection with speed v^*, where

$$v^* = V\left(\frac{g}{g + kV^2}\right)^{\frac{1}{2}}.$$

[6]

QUESTION 4

(a) Show, by means of the substitution $r = 1/u$, that the equation of a particle in a central field is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{mh^2u^2}.$$

[12]

(b) A comet moves in a plane under the gravitational attraction of the sun, which is situated at the origin O. Given that the attractive force between the sun and the comet can be written as

$$f(r) = -\frac{GMm}{r^2};$$

(i) Show that

$$(\ddot{r} - r\dot{\theta}^2) = -\frac{GM}{r^2},$$

$$r^2\dot{\theta} = h,$$ [1]

where r and θ are plane polar coordinates, h is a constant, G is the gravitational constant, M is the mass of the sun, and m is the mass of the comet.

(ii) Suppose that at the initial instant, $\theta = 0$, the comet is at distance d from the sun and is moving with speed v in a direction perpendicular to the radius vector from the sun. Show, by means of the substitution $r = \frac{1}{u}$, that the equation of motion of the particle is

$$\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = \frac{GM}{d^2v^2}.$$

[4]

(c) A particle moves in a central force field defined by

$$\mathbf{F} = \frac{-K}{r^2}\hat{\mathbf{r}},$$

where K is a constant.

(i) Find the potential energy of the particle. [2]

(ii) Find the work done in moving the particle from a point on the circle of radius $r = a$ to a point on the circle of radius $r = b$, where $a, b > 0$. [1]

QUESTION 5

(a) Describe two physical systems that can be modeled by the differential equation

$$\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + w_0^2 x = f(t),$$

where k and w_0 are fixed constants and $f(t)$ corresponds to an external force. [4]

(b) A 20 kg weight suspended at the end of a vertical spring stretches it 20 cm. Assuming no external forces, find the position of the weight at any time t if initially the weight is

(i) pulled down 10 cm and released,

(ii) pulled down 15 cm and given an initial speed of 105 cm/sec downward.

Find the period and the amplitude in each case. [8]

(c) Solve the mass-spring problem in (b) taking into account an external damping force given in kilograms by $v/7$, where v is the instantaneous speed in cm/sec. [8]

QUESTION 6

(a) If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$ and $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C})\mathbf{A}$ show that $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d}) = (\mathbf{a} \cdot \mathbf{c})(\mathbf{b} \cdot \mathbf{d}) - (\mathbf{a} \cdot \mathbf{d})(\mathbf{b} \cdot \mathbf{c})$. [5]

(b) Given the points $A(2,3,1)$, $B(-1,1,2)$ and $C(1,-2,3)$:

(i) Show that the acute angle θ which the median to the side AC makes with the side BC is given by

$$\theta = \cos^{-1}(\frac{\sqrt{91}}{14});$$

[6]

(ii) Find the angle between $\overline{AB}$ and $\overline{BC}$; [3]

(iii) Find the equation of the plane passing through the three points. [6]

QUESTION 7

(a) An inductor of 2 henries, a resistor of 16 ohms, and a capacitor of 0.02 farads are connected in series with a battery of e.m.f. $E = 100\sin(3t)$. At $t \leq 0$ the charge on the capacitor and the current in the circuit are zero. Find the charge and current at any time $t > 0$. [12]

(b) A particle of mass 2 gm moves along the $x-$ axis attracted toward the origin O with a force numerically equal to $8x$. If it is initially at rest at $x = 10$ cm, find its position at any later time t assuming

(i) no other forces act on the particle, [4]

(ii) a damping force numerically equal to 8 times the instantaneous speed acts on the particle. [4]

END OF EXAMINATION