UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2009/2010

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

The position vector $\mathbf{r}$ of a moving point is given by

$$\mathbf{r} = 3\cos(3t)\hat{\mathbf{i}} + 3\sin(3t)\hat{\mathbf{j}} + (9t - 6)\hat{\mathbf{k}}.$$

Find,

(a) the velocity vector $\mathbf{v}$ [1]

(b) the acceleration vector $\mathbf{a}$ [1]

(c) the speed [2]

(d) the unit tangent vector $\widehat{\mathbf{T}}$ [2]

(e) the curvature [5]

(f) the unit normal vector $\widehat{\mathbf{N}}$ [2]

(g) the unit binormal vector $\widehat{\mathbf{B}}$ [3]

(h) the tangential component of acceleration [2]

(i) the normal component of acceleration [2]

QUESTION 2

(a) A particle starts from rest and moves in a straight line with acceleration $(16-2v^2)$, where v is its speed. Show that the particle has terminal velocity $V = \sqrt{8}$, and find an expression for v in terms of the distance traveled. [6]

(b) A body of unit mass moving in a straight line is projected with speed u from a point at a distance d from the origin. It is acted upon by a force $\frac{k}{x}$, where k is a constant and x is the distance from the origin. Show that

$$x = de^{\frac{u^2-v^2}{2k}},$$

where v is the body's speed. [5]

(c) A particle drops from rest under gravity in a medium which exerts a resistive force of kv per unit mass, where k is a constant and v is the speed. Show that the terminal velocity is given by

$$V = \frac{g}{k}.$$

Also show that the speed v and the distance traveled x at any time t are given by

$$v = V\left(1 - e^{\frac{-gt}{V}}\right)$$

and

$$x = Vt - \left(\frac{V^2}{g}\right)\left(1 - e^{\frac{-gt}{V}}\right).$$

[9]

QUESTION 3

(a) A particle of mass m moves in a central force field $\mathbf{F} = \frac{K}{r^n}\hat{\mathbf{r}}$ where K and n are constants. It starts from rest at $r = a$ and arrives at $r = 0$ with finite speed v_0. Prove that

$$v_0 = \left\{ \frac{2Ka^{1-n}}{m(n-1)} \right\}^{1/2}.$$

[8]

(b) Prove that

$$\nabla \cdot (\phi \mathbf{A}) = (\nabla \phi) \cdot A + \phi(\nabla \cdot \mathbf{A}).$$

[7]

(c) If $\phi = x^2yz^3$ and $\mathbf{A} = xz\mathbf{i} - y^2\mathbf{j} + 2x^2y\mathbf{k}$, find

$$div(\phi \mathbf{A}).$$

[5]

QUESTION 4

A projectile of mass m is launched with initial speed U at an angle θ with the horizontal. If the projectile has acting upon it a force due to air resistance equal to $-\beta\mathbf{v}$, where β is a positive constant and $\mathbf{v}$ is the instantaneous velocity, prove that the position at any time is given by

$$\mathbf{r} = \frac{mU}{\beta}(\cos\theta\mathbf{j} + \sin\theta\mathbf{k})(1 - e^{-\beta t/m}) - \frac{mg}{\beta}(t + \frac{m}{\beta}e^{-\beta t/m} - \frac{m}{\beta})\mathbf{k}.$$

[20]

QUESTION 5

(a) Show, by means of the substitution $r = 1/u$, that the equation of the particle in a central field is

$$\frac{d^2u}{d\theta^2} + u = -\frac{f(1/u)}{mh^2u^2}.$$

[7]

(b) Suppose that a particle mass m is acted upon by a force $\alpha r^{-2} + \beta r^{-3}$ per unit mass where ($\beta = \frac{1}{2}\alpha a$) directed towards the origin $r = 0$ of an inertial frame. Suppose, also, that at $\theta = 0$ and $t = 0$, measurements of distance and velocity of the particle show that it is at distance a from the origin moving with velocity $\sqrt{\alpha/a}$ in a direction perpendicular to the radius vector. If $u = 1/r$, prove that

$$u = \frac{2}{a} - \frac{1}{a}\cos\frac{\theta}{\sqrt{2}}.$$

[13]

QUESTION 6

The following equation could represent the damped vertical motion of a mass supported by a spring and subjected to an external force:

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \frac{\mathrm{d}x}{\mathrm{d}t} + 36x = 10\cos(wt), \quad \text{for } t > 0,$$

the system being in equilibrium under no force for $t \leq 0$.

(a) Find the period of the free oscillations. [5]

(b) Obtain expressions in terms of w for the amplitude and phase of the forced oscillation. [7]

(c) Find the condition for resonance. [4]

(d) Plot the curve of the amplitude against w for a range $4 \leq w \leq t$. [4]

QUESTION 7

(a) Express $x = -3\cos(2t - \frac{\pi}{2})$ in standard form. [2]

(b) State whether x leads or lags y, and by how much, in the equations $x = -3\cos(2t)$, $y = 4\cos(2t)$. [4]

(c) A 20 kg weight suspended at the end of a vertical spring stretches it 20 cm. Assuming no external forces, find the position of the weight at any time t if initially the weight is

(i) pulled down 10 cm and released,

(ii) pulled down 15 cm and given an initial speed of 105 cm/sec downward.

Find the period and the amplitude in each case. [6]

(d) Solve the mass-spring problem in (c) if an external force given by $F(t) = 20\cos 7t$ is applied for $t > 0$. Give a physical interpretation of what happens as t increases. [8]

END OF EXAMINATION