UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2009/2010

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. Let $f(x) = x - e^x$

   (a) Show that $f(x)$ has exactly one root in [0, 1]. [5 marks]

   (b) Compute an approximation to the root by taking 4 steps of the bisection method. [6 marks]

   (c) How many iterations would be required to locate this zero to a tolerance of 10^{-5}? [3 marks]

   (d) Compute an approximation to the root by taking 3 steps of the Newton's method starting with $x_0 = 0.5$. [6 marks]

QUESTION 2

2. (a) Suppose that $f(-1) = 3$, $f(0) = 4$ and $f(2) = 5$. Find the Lagrange interpolating polynomial which interpolates these values, and use it to estimate $f'(0)$. [10 Marks]

   (b) Given the data

x	1	$\frac{3}{2}$	0	2
$f(x)$	3	$\frac{13}{4}$	3	$\frac{5}{3}$

   (i) construct a divided difference table. [5 marks]

   (ii) write down the Newton form of the interpolating polynomial. [5 marks]

QUESTION 3

3. (a) Convert the 32-bit floating-point number 0 0111 1010 0101 0100 0000 0000 0000 000 to its decimal equivalent. [6 marks]

   (b) Given the function $f(h) = \sqrt{9-h} - 3$

   (i) find a suitable function $g(h)$ that has been reformulated to be algebraically equivalent to $f(h)$ with the aim of avoiding loss of significance error. [3 marks]

   (ii) Compare the results of calculating $f(0.0001)$ and $g(0.0001)$ using **six digits** and **chopping**. [4 marks]

   (c) For the scheme $x_{n+1} = x_n + c(x_n^2 - 7)$, find the range of values of c for which convergence to the positive fixed point is guaranteed. For what value of c is convergence quadratic? [7 marks]

QUESTION 4

4. (a) Establish a numerical integration formula of the form

$$\int_a^b f(x)\,dx \approx w_1 f(a) + w_2 f(b)$$

that is accurate for polynomials of as high a degree as possible. [8 marks]

(b) Use the **Gaussian elimination** procedure to compute the LU factorization of the matrix

$$A = \begin{pmatrix} 2 & 2 & -1 \\ 4 & 5 & 2 \\ -2 & 1 & 2 \end{pmatrix}$$

[12 marks]

QUESTION 5

5. (a) Evaluate the integral

$$\int_0^2 \ln(1+x)\,dx$$

by the Trapezoidal rule with accuracy $\epsilon = 0.05$ [10 Marks]

(b) Convert the binary number $(0.111111...)_2$ to its decimal equivalent [5 Marks]

(c) Convert the decimal number $\frac{43}{5}$ to its binary equivalent. [5 Marks]

QUESTION 6

6. Suppose we know the following values of a function f:

$$f(0) = 1, f(1) = 2, f(2) = 8$$

(a) Evaluate the divided-differences $f[0], f[0,1], f[0,1,2]$. [4 marks]

(b) Evaluate the forward-differences $\Delta f(x_0), \Delta^2 f(x_0)$. [4 marks]

(c) Write down the appropriate Newton's interpolating polynomial. [2 marks]

Use LU decomposition to solve the following linear system

$$\begin{aligned} x + 2y + 3z &= 4 \\ x - y + 6z &= -1 \\ 2x + y &= 0 \end{aligned}$$

[10 marks]

7. QUESTION 7

7. (a) Use the two-point Gaussian Quadrature rule,

$$\int_{-1}^{1} f(x)\, dx \approx f\left(\frac{-\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right),$$

to approximate the integral

$$\int_0^1 x^2 e^{-x}\, dx$$

and compare your result against the exact value of the integral. [10 marks]

(b) For the data

x	- 2	- 1	0	1	2	3
$f(x)$	15	5	1	3	11	25

(i) Construct the foward difference table. [7 marks]

(ii) Use the resulting polynomial to find $f(0.5)$. [3 marks]