UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2009/10

BSc./B.Ed./B.A.S.S III

TITLE OF PAPER	:	DYNAMICS II
COURSE NUMBER	:	M355
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Give the definitions and examples of

(i) degrees of freedom,

(ii) generalized coordinates,

(iii) holonomic/nonholonomic systems,

(iv) reonomic/scleronomic systems

(v) conservative/nonconservative systems. [1,1,2,2,2]

(b) Prove the interchange of d and ∂ lemma

$$\frac{d}{dt}\left(\frac{\partial \overline{r}_\nu}{\partial q_i}\right) = \frac{\partial \dot{r}_\nu}{\partial q_i}$$

[7]

(c) Consider a mass m suspended on a spring of negligible mass and stiffness c. Derive Lagrange's equation. Compare with Newton's mechanics equation. [5]

QUESTION 2

(a) Derive Lagrange's equations for the holonomic,scleronomic system with n degrees of freedom. Hint: You may use results from Q1 (b) and cancellation of dot property lemma. [10]

(b) A mass m_1 is suspended to a spring of stiffness c_1. To this mass another mass m_2 is suspended on a spring of stiffness c_2. Neglect with the masses of the springs. Derive Lagrange's equations [10]

QUESTION 3

(a) Let potential energy be a function of both generalized coordinates and velocities. Show that

$$T + \Pi - \sum_{i-1}^{n} \dot{q}_i \frac{\partial \Pi}{\partial \dot{q}_i} = const,$$

in the usual notations [12]

(b) Give the definition of;

(i) generalized momenta,

(ii) cyclic coordinate,

(iii) Hamiltonian. [1,1,1]

(c) Derive Hamilton's equations for the mathematical pendulum. [5]

QUESTION 4

(a) Derive Hamilton's equations in the general case when Hamiltonian contains the time explicitly. [8]

(b) Consider a system with two degrees of freedom with kinetic and potential energies as follows

$$T = \frac{1}{2}m(\dot{r}^2 + r^2 \sin^2 \alpha \dot{\varphi}^2), \quad \Pi = mgr\cos\alpha$$

where α is a constant. Derive Hamilton's equations. [12]

QUESTION 5

(a)

(i) Give three conditions for the canonical transformation.

(ii) For what values of the constant parameters a and b the following transformation is canonical?

$$Q = q^a \cos bp, \quad P = q^a \sin bp.$$

[3,5]

(b) Consider a dynamic variable $F(q, p, t)$. Let H be a Hamiltonian of a system. Show that

$$\frac{dF}{dt} = \frac{\partial F}{\partial t} + [F, H].$$

[6]

(c) Use poisson brackets to show that the following transformation is canonical

$$Q_1 = \sqrt{2p_1}\sin q_1 + p_2, \qquad P_1 = \frac{1}{2}\left(\sqrt{2p_1}\cos q_1 - q_2\right),$$

$$Q_2 = \sqrt{2p_1}\cos q_1 + q_2, \qquad P_2 = -\frac{1}{2}\left(\sqrt{2p_1}\sin q_1 - p_2\right).$$

[6]

QUESTION 6

(a) Consider a functional

$$V[y(x)] = \int_{x_0}^{x_1} F(x, y(x), \;\; y'(x))dx$$

subject to boundary conditions

$$y(x_0) = y_0, \quad y(x_1) = y_1$$

Show that if $y(x)$ is an extremal then it satisfies the Euler equation. [8]

(b) Find the extremals for the functional

$$V[y(x)] = \int_0^1 \left[(y')^2 + 12xy\right] dx, \quad y(0) = 0, \quad y(1) = 1.$$

[4]

(c) Let F in (a) have form

$$F(x, y, y') = M(x, y) + N(x, y)y'.$$

(i) Show that $\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x} = 0$ for extremal solutions.

(ii) Consider a functional

$$V[y(x)] = \int_0^1 (y^2 + x^2 y')dx, \quad y(0) = 0, \quad y(1) = a.$$

for which a there is an extremal solution?

[4,4]

QUESTION 7

(a) Find the extremals for the following functionals

(i) $V[y(x), z(x)] = \int_0^{\frac{\pi}{2}} \left[(y')^2 + (z')^2 + 2yz\right] \; dx,$

$y(0) = 0, \;\; y\left(\frac{\pi}{2}\right) = 1, \;\; z(0) = 0, \;\; z\left(\frac{\pi}{2}\right) = -1.$

(ii) $V[y(x)] = \int_0^1 \left[1 + (y'')^2\right] \; dx,$

$y(0) = 1, \;\; y'(0) = 0, \;\; y(1) = 1, \;\; y'(1) = 1.$

[7,7]

(b) Find Ostrogradski's equation for the following functional

$$V[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 2zf(x,y)\right] dxdy.$$

where z is known on the boundary of region D. [6]