University of Swaziland

Final Examination, December 2010

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the exact value of:

i. $^{-}410 + {}^{-}403 + {}^{-}396 + \cdots + 423 + 430$ [5]

ii. $2 - 6 + 18 - 54 + \cdots + 9565938 - 28697814$ [5]

(b) Evaluate and express in the form $a + ib$.

i. $\dfrac{1 + 2i^{12} - 3i^{17}}{3i^{21} + 2i^{71} - 1}$ [5]

ii. $\left(i - \sqrt{3}\right)^{8}$ [5]

Question 2

(a) Describe the locus of points represented by the given equation. Make a sketch of the curve.

i. $4x^2 + 4y^2 + 80x + 12y + 265 = 0$ [4]

ii. $y^2 + 4y + 20x + 4 = 0$ [4]

(b) Evaluate

$$\begin{vmatrix} 1 & 2 & 0 & -3 \\ -2 & 0 & 4 & -1 \\ 2 & 1 & -4 & 0 \\ 0 & -1 & 0 & 3 \end{vmatrix}$$ [6]

(c) Prove

$$\frac{\sin A + \tan A}{\csc A + \cot A} = \sin A \tan A.$$ [6]

Question 3

(a) Find the middle term of the binomial expansion of

$$\left(\frac{y\sqrt{y}}{x} - \frac{x^2}{\sqrt{y}}\right)^{24}. \qquad [6]$$

(b) Find the value of

$$\sum_{n=1}^{\infty} \left(\frac{4}{5}\right)^n. \qquad [4]$$

(c) Given that $z = i - 2$ is a root of

$$z^4 - 2z^3 - 6z^2 + 22z + 65,$$

find the 3 other roots. [10]

Question 4

(a) Solve for x :

i. $\log_8\left(\log_7 x^2\right) = \frac{1}{3}$ [4]

ii. $e^x + e^{-x} = \frac{5}{2}$ [6]

(b) Use mathematical induction to prove

$$1 + r + r^2 + \cdots + r^{n-1} = \frac{1 - r^n}{1 - r}, \quad r \neq 1, \ n \geqslant 1. \qquad [10]$$

Question 5

(a) State the *Rational Root Theorem.* [5]

(b) Use Cramer's rule to solve

$$\begin{aligned} x - 2y + 3z &= -4, \\ 2x + 3y - z &= -1, \\ 3x - y + 2z &= -9. \end{aligned}$$

[15]

Question 6

(a) Divide

$$\frac{2x^5 + 7}{x^2 - 2}.$$ [7]

(b) Find the the first 4 terms of the binomial expansion of

$$\left(\frac{1}{a^2} - 2ab\right)^{-\frac{1}{2}}.$$ [7]

(c) Find all the fourth roots of -16. [6]

Question 7

(a) State the *Remainder Theorem.* [4]

(b) Solve for x (in the range $0 \leqslant x < 2\pi$)

$$\cos 2x + \cos x + 1 = 0.$$ [6]

(c) Prove that

$$P(n) = n(n^2 + 2), \quad n \geqslant 1$$

is always divisible by 3. [10]