University of Swaziland

Supplementary Examination, July 2011

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. and Analytic Geometry

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the binomial exapnsion of

$$\left(\frac{x^2}{y} - \frac{2y}{x}\right)^5$$

and simplify term by term. [7]

(b) Divide

$$\frac{x^4 - x^3 - x^2 + 1}{x + 1}.$$ [6]

(c) Solve for x (in the range $0 \leqslant x < 2\pi$)

$$\sin 2x + \sin x = 0.$$ [7]

Question 2

(a) Find the sum of all multiples of 3 between 9 and 900 inclusive. [6]

(b) Find all root (real and/or complex):

$$x^4 - 16 = 0.$$ [6]

(c) Evaluate and express your answer in the form $a + ib$.

$$2i + \frac{50}{(1 - 2i)^2}.$$ [8]

Question 3

(a) Evaluate

$$\begin{vmatrix} 1 & -1 & 2 & -1 \\ 0 & 1 & 0 & 1 \\ -2 & 2 & 1 & 3 \\ 0 & 3 & -4 & 0 \end{vmatrix}. \qquad [7]$$

(b) Prove

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta. \qquad [7]$$

(c) Solve

$$\log_3(8x+1) - 2 = \log_3(x-3). \qquad [6]$$

Question 4

(a) Find the first four terms of the binomial expansion of

$$\frac{1}{(1+2x)^2}. \qquad [4]$$

(b) Use mathematical induction to prove

$$1+3+5+\cdots+(2n-1) = n^2, \quad n \geqslant 1. \qquad [8]$$

(c) Given that $\sin A = \frac{12}{13}$ and A is in QI, find

i. $\sin 2A$ [3]

ii. $\cos 2A$ [3]

Hence state, with reasons, the quadrant in which the angle $2A$ lies. [2]

Question 5

Use Cramer's rule to solve

$$\begin{aligned} x + 2y + 3z &= 4, \\ 2x + 3y + z &= 1, \\ 3x + y + 2z &= -5. \end{aligned}$$

[20]

Question 6

(a) Solve

$$2^{x-2} = 3 \cdot 5^{-x}.$$ [4]

(b) The sum of the second and third terms of a GP is 12. If the sum of the third and fourth terms is -36, find the sum of the first 20 terms. [8]

(c) Describe the locus of points defined by the given equation. Hence make a sketch of the curve.

$$y^2 + 4y + 20x - 56 = 0.$$ [8]

Question 7

(a) State *de Moivre's Theorem.* [4]

(b) Find the term independent of x in the binomial expansion of

$$\left(\frac{x^2}{y} - \frac{y}{x}\right)^{21}.$$ [6]

(c) Use mathematical induction to prove that

$$\cos n\pi = (-1)^n, \quad n \in \mathbb{Z}.$$ [10]