University of Swaziland

Final Examination, May 2011

BSc I, EEng I, BEd I

Title of Paper : Introduction to Calculus
Course Number : M115
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate

i. $\lim_{x\to 0}\left(\dfrac{1-\sqrt{1+x}}{x}\right)$ [5]

ii. $\lim_{x\to 2}\left(\dfrac{x^2-4}{x^2+4}\right)$ [5]

(b) Find y'

i. $y=\left(x^2+1\right)^{-4x}$ [5]

ii. $e^{xy}=\ln(x+y)$ [5]

Question 2

(a) Evaluate each integral.

i. $\int\left(4-6x^2-\dfrac{6}{x^2}-\dfrac{6}{x}-\dfrac{6}{\sqrt{x}}-\dfrac{6}{e^{2x}}\right)\mathrm{d}x$ [5]

ii. $\int_0^{\frac{\pi}{2}}\dfrac{\sin 2\theta}{1+\sin^2\theta}\mathrm{d}\theta$ [5]

(b) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x)=\sqrt{5+4x}.$$ [10]

Question 3

(a) Use Leibniz rule to find $\dfrac{\mathrm{d}^4y}{\mathrm{d}x^4}$ given that

$$y=\left(x^2-x\right)e^{2x}.$$ [8]

(b) Evaluate

$$\int\left(x^2-x\right)e^{2x}\mathrm{d}x.$$ [8]

(c) Find the equation of the tangent to the curve $y = x^3 - 3x^2 - 9x + 5$ at $(0, 5)$. [4]

Question 4

(a) Differentiate and simplify

$$y = \tan^{-1}\left(\frac{1}{x}\right) + \tfrac{1}{2}\ln\left(x^2 + 1\right). \quad [8]$$

(b) Evaluate

i. $\displaystyle\int_0^{\frac{\pi}{4}} \sin^3\theta\cos\theta \mathrm{d}\theta$ [6]

ii. $\displaystyle\int \tan\theta\sec^4\theta \mathrm{d}\theta$ [6]

Question 5

(a) Consider the function

$$f(x) = 10 - 4x^2 + 4x^3 - x^4.$$

i. Find the intervals/points where the graph of $f(x)$ is increasing, decreasing or stationary [6]

ii. Classify the stationary point(s) [3]

iii. Find the intervals/points where the graph of $f(x)$ is concave up or down [3]

iv. Find the inflexion point(s) of $f(x)$ (if any) [2]

v. Make a sketch of the graph of $y = f(x)$. [6]

Question 6

Integrate

a. $\int \frac{5x-1}{2x^2+x-10}\mathrm{d}x$ [14]

b. $\int \frac{3x-2}{\sqrt{4+4x-3x^2}}\mathrm{d}x$ [6]

Question 7

(a) Differentiate and simplify

$$y = \frac{\cos\theta}{\sin\theta - \cos\theta}.$$ [8]

(b) Find the *exact* value of the area bounded by the parabola $y = x^2 - x - 1$ and the straight line $y = x + 1$. [12]