University of Swaziland

Supplementary Examination, July 2011

BSc I, EEng I, BEd I

Title of Paper : Introduction to Calculus
Course Number : M115
Time Allowed : Three (3) hours
Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%..
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate

i. $\lim_{x\to 2}\left(\frac{4-x^2}{x^2-5x+6}\right)$ [5]

ii. $\lim_{x\to \infty}\left(\frac{4-x^2}{x^2-5x+6}\right)$ [5]

(b) Find y'

i. $y = x^{1-2x}$ [5]

ii. $y = \cos(x-y)$ [5]

Question 2

(a) Evaluate each integral.

i. $\int\left(4-2x^3-\frac{3}{2x^3}-6\sqrt{x}-\cos 2x\right)\mathrm{d}x$ [5]

ii. $\int_0^1 \frac{3x^3}{1+x^4}\mathrm{d}x$ [5]

(b) Use the limit definition of the derivative to find $f'(x)$ if

$$f(x) = \frac{2}{x}.$$ [10]

Question 3

(a) Use Leibniz rule to find $\frac{\mathrm{d}^4y}{\mathrm{d}x^4}$ given that

$$y = x^3 \ln x.$$ [10]

(b) Evaluate

$$\int 54x^2e^{-3x}\mathrm{d}x.$$ [10]

Question 4

(a) Differentiate and simplify

$$y = \frac{1}{x+1} + \ln\left(x^2 + 2x + 1\right). \qquad [10]$$

(b) Evaluate

$$\int_0^3 \frac{dx}{9 + x^2} \qquad [10]$$

Question 5

(a) Consider the function

$$f(x) = 3 - 3x^2 + x^3.$$

i. Find the intervals/points where the graph of $f(x)$ is increasing, decreasing or stationary [6]

ii. Classify the stationary point(s) [3]

iii. Find the intervals/points where the graph of $f(x)$ is concave up or down [3]

iv. Find the inflexion point(s) of $f(x)$ (if any) [2]

v. Make a sketch of the graph of $y = f(x)$. [6]

Question 6

Integrate

a. $\int \frac{2x-3}{(x+4)(x-3)}\mathrm{d}x$ [10]

b. $\int \sin^4\theta \mathrm{d}\theta$ [10]

Question 7

(a) Differentiate and simplify

$$y = \frac{4x}{2-3x^2}.$$ [8]

(b) Find the *exact* value of the area bounded by the parabola $y = 8 - x^2$ and the straight line $y = 2 - x$. [12]