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University of Swaziland

Final Examination, 2010/2011

BSc II, Bass II, BEd II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

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(a) Consider the function $f(x) = x^4 - 2x^2 + 3$.

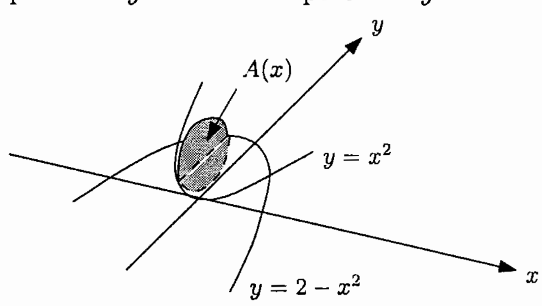
i. Find the intervals on which f is increasing or decreasing. [4]

ii. Find the local maximum and local minimum values of f. [3]

iii. Find the intervals on which f is concave up and concave down. [4]

iv. Find the inflection points of f. [2]

(b) Find the volume of the solid that lies between the planes $x = -1$ and $x = 1$ and whose cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$ (see below). [7]



QUESTION 2

Evaluate the following limits.

(a) $\lim_{x \to 1} \frac{\ln x}{x - 1}$ [4]

(b) $\lim_{x \to \infty} x \sin\left(\frac{\pi}{x}\right)$ [5]

(c) $\lim_{x \to (\frac{\pi}{2})^-} (\sec x - \tan x)$ [5]

(d) $\lim_{x \to 0} (1 - 2x)^{1/x}$ [6]

QUESTION 3

(a) i. State (do not prove) the Second Derivative Test for local extrema. [4]

ii. Find the local extrema for the function $f(x) = x^3 - 3x^2 + 3$. [4]

(b) Find the absolute maximum and absolute minimum values of the function $f(x) = \frac{x}{x^2 + 1} + 1$ on the interval $[0, 2]$. [5]

(c) Use cylindrical shells to find the volume of the solid obtained when the region bounded by the curve $y = \sqrt{x}$, the line $x = 4$ and the x-axis is rotated about the x-axis. [7]

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QUESTION 4

(a) The region bounded by the graph of $y = x^2$, the lines $x = 1$ and $x = 2$ and the x-axis is rotated about the x-axis to generate a solid. Find the volume of the solid. [5]

(b) Find the length of the curve $y = 2x^{3/2}$ between $x = 0$ and $x = 3$. [5]

(c) Find the *area* of the surface generated by revolving the curve $y = 2\sqrt{x}$, $1 \leq x \leq 2$ about the x-axis. [10]

QUESTION 5

(a) Find the length of the curve with parametric equations $x = \cos t$, $y = t + \sin t$, $0 \leq t \leq \pi$.
[***Hint:*** $1 + \cos t = 2\cos^2\left(\frac{t}{2}\right)$.] [10]

(b) The region bounded by the curves $y = x$ and $y = \sqrt{x}$ is rotated about the line $y = 1$ to generate a solid. Find the volume of the solid. [10]

QUESTION 6

(a) Investigate the convergence of each series.

i. $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$ ii. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)$ [5,5]

(b) Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$. [10]

QUESTION 7

(a) Consider the sequence $\{a_n\}$ defined recursively by

$$a_1 = 2 \qquad a_{n+1} = \tfrac{1}{2}(a_n + 6) \quad \text{for } n = 1, 2, 3, \ldots$$

i. Use mathematical induction to show that $a_{n+1} > a_n$ for all $n \geq 1$. [4]

ii. Use mathematical induction to show that $a_n < 6$ for all n. [4]

iii. Use your answers to i. and ii. to determine whether or not the sequence is convergent. [4]

(b) Show that the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. [8]

END OF EXAMINATION PAPER