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University of Swaziland

Supplementary Examination, 2010/2011

BSc II, Bass II, BEd II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

Consider the function

$$f(x) = \frac{x^2 - 3x}{(x+3)^2}.$$

(a) Show that

$$f'(x) = \frac{9x - 9}{(x+3)^3} \quad \text{and} \quad f''(x) = \frac{54 - 18x}{(x+3)^4}$$

(2,2)

(b) What is the domain of f? (2)

(c) Find the critical points of f. (3)

(d) Find the intervals on which f is increasing or decreasing. (4)

(e) Find the local maximum and minimum values of f. (2)

(f) Find the intervals on which f is concave up and the intervals on which f is concave down. (3)

(g) Find inflection points of f if any. (2)

QUESTION 2

Evaluate the following limits.

(a) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$ [4]

(b) $\lim_{x \to \infty} x \sin\left(\dfrac{1}{x}\right)$ [4]

(c) $\lim_{x \to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln x}\right)$ [6]

(d) $\lim_{x \to \infty} x^{2/x}$ [6]

QUESTION 3

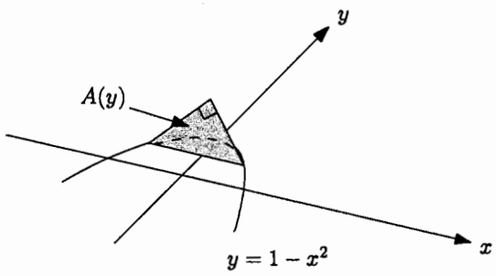
(a) Use cylindrical shells to find the volume of the solid obtained when the region bounded by the curve $y = 3x - x^2$ and the x-axis is rotated about the vertical line $y = -1$. [10]

(b) Find the length of the curve with parametric equations $x = \cos t$, $y = t + \sin t$, $0 \leq t \leq \pi$. [10]

[***Hint***: $1 + \cos t = 2\cos^2\left(\dfrac{t}{2}\right)$.]

QUESTION 4

(a) Find the volume of the solid whose base is a plane region R enclosed by the x-axis and the curve $y = 1 - x^2$. Each cross-section of the solid perpendicular to the y-axis is an isosceles right triangle with hypotenuse lying in R (see below). [7]



(b) The region bounded by the graph of $y = x^2$, the lines $x = 1$ and $x = 2$ and the x-axis is rotated about the x-axis to generate a solid. Find the volume of the solid. [7]

(c) Find the Maclaurin series for $\cos x$. [6]

QUESTION 5

(a) Find the length of the curve $y = \frac{2}{3}(x-4)^{3/2}$ between $x = 7$ and $x = 12$. [5]

(b) The line segment $x = 1 - y$, $0 \leq y \leq 1$ is rotated about the y-axis to generate an open cone. Find the *surface area* of the cone generated. [10]

(c) Find all local extrema for the function $f(x) = x^4 - 6x^2$. [5]

QUESTION 6

(a) Investigate the convergence of each series.

i. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2}{5n^2+4}$ ii. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$. [4,6]

(b) Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$. [10]

QUESTION 7

(a) Show that the sequence defined by

$$a_1 = 2, \qquad a_{n+1} = \tfrac{1}{2}(a_n + 6), \qquad \text{for } n = 1, 2, 3, \ldots$$

is increasing and $a_n < 6$ for all n. Deduce that $\{a_n\}$ is convergent and find its limit. [10]

(b) Show that the p-series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. [10]

END OF EXAMINATION PAPER