University of Swaziland

Supplementary Examination, July 2011

BSc II, EEng II, BEd II, BASS II

Title of Paper : Calculus II

Course Number : M212

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

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Question 1

(a) Use the Lagrange multpliers to find the maximum and minimum values of the function

$$f(x, y) = xy$$

subject to the constraint

$$\frac{x^2}{8} + \frac{y^2}{2} = 1.$$

[10]

(b) Find the volume of the solid of revolution if the area bounded by the curve

$$y = 2x - x^2 \quad \text{and} \quad y = x^2 - 2x$$

is rotated about the y-axis. [8]

(c) Express in polar form

$$y^2 = 1 - 4x.$$ [2]

Question 2

(a)

i. Sketch the graph of the curve

$$r = 4 + 3\cos\theta$$

ii. Find the area enclosed by the curve.

(b) Use the chain rule to find $\frac{\partial z}{\partial r}$ and $\frac{\partial z}{\partial s}$ where

$$z = x^2 \sin y, \quad x = r^2 + s^2, \quad y = 2rs.$$

(c) Find the volume of the solid whose base in the region in the xy-plane bounded by the curves $y = x^2 - x$ and $y = x$ and whose top is bounded by $z = 5x^2$.

Question 3

(a) Assuming that the equation

$$xy^2z^3 + x^3y^2z = x + y + z$$

defines z implicitly as a function of x and y, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$. [8]

(b) Find the total differential of

$$f(x, y, z) = \frac{xy}{z}.$$ [4]

(c) Evaluate the iterated integral

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} dx dy.$$ [8]

Question 4

(a) Evaluate the iterated integral by first converting to polar co-ordinates.

$$\int_0^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} x^2y^2 dx dy.$$ [8]

(b) Find the directional derivative of the function

$$f(x, y, z) = x\sin(yz)$$

at the point $(1, 3, 0)$ in the direction of the vector $v = \hat{\imath} + 2\hat{\jmath} - \hat{k}$. [8]

(c) Express in rectangular form

$$r = 2 - \cos\theta. \qquad [4]$$

Question 5

(a) Locate all the relative extrema and saddle points of the function

$$f(x, y) = xy - x^2y - xy^2. \qquad [10]$$

(b) Find the surface area of the the portion of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$. [10]

Question 6

(a) Find the length of the curve

$$r = 1 - \cos\theta, \quad 0 \leqslant \theta \leqslant \pi. \qquad [7]$$

(b) Let $z = f(r - s)$, where $f(x)$ is a function of x with a continuous derivative, show that

$$\frac{\partial z}{\partial r} + \frac{\partial z}{\partial s} = 0. \qquad [5]$$

(c) Evaluate

$$\iint_R \frac{1}{2(x-3)} dxdy$$

where R is the region between the curves $y = x^2$ and $y = -x^2 + 6x$. [8]

Question 7

(a) Prove that

$$f(x,y) = \tan^{-1}\left(\frac{2x}{x^2 - y^2}\right)$$

is harmonic. [6]

(b) Find the rectangular equation of the line tangent to the curve $r = 3 + \cos 2\theta$ at the point $(3, \frac{3}{4}\pi)$. [5]

(c) Evaluate

$$\iiint_R z\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$$

where R is the solid in the first octant bounded by the cylinder $x^2 + y^2 = 4$ and the plane $x + y = 3$. [9]