University of Swaziland

Final Examination, May 2011

BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. **Submit solutions to ONLY FIVE questions.**
4. Show all your working.
5. A Table of Laplace Transforms is provided at the end of the question paper.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Solve

$$y' = \frac{2 + ye^{xy}}{2y - xe^{xy}}$$

[8 marks]

(b) Show that the solution for the linear differential equation

$$p(x)y'(x) + q(x)y(x) = r(x)$$

is given by

$$y(x) = e^{-\int \frac{q(x)}{p(x)}dx}\left(\int \frac{r(x)}{p(x)} e^{\int \frac{q(x)}{p(x)}dx} + c\right).$$

Hence solve

$$xy' + 2y = 4x^2.$$

[12 marks]

Question 2

Find the general solution of the following differential equations

(a)

$$y' = \frac{2y^4 + x^4}{xy^3}$$

[10 marks]

(b)

$$xydx + (1 + x^2)dy = 0$$

[10 marks]

Question 3

(a) Given that the differential equation

$$\mu(y)M(x,y)dx + \mu(y)N(x,y)dy = 0$$

is exact. Show that

$$\mu(y) = e^{\int \frac{N_x - M_y}{M}dy}.$$

[8 marks]

(b) Find the second solution of

$$2x^2y'' + xy' - 3y = 0$$

given that $y_1 = x^{-1}$ is a solution. Show that this solution, along with the given solution form a fundamental set of solutions for the differential equation. [12 marks]

Question 4

Find the series solution of

$$(x^2 + 1)y'' - 4xy' + 6y = 0$$

about $x = 0$. [20 marks]

Question 5

(a) The differential equation

$$y''' + 2y'' - y' - 2y = e^x + x^2$$

has

$$y_c = c_1e^x + c_2e^{-x} + c_3e^{2x}$$

as the complementary solution. Find the particular solution for the differential equation. [7 marks]

(b) Solve using Laplace transforms

(i) $\dot{y}(t) - 5y(t) = e^{5t}, \quad y(0) = 0.$ [5 marks]

(ii) $\ddot{y}(t) + 16y(t) = 2\sin 4t, \quad y(0) = -\frac{1}{2}, \quad \dot{y}(0) = 0.$ [8 marks]

Question 6

(a) Use two methods to solve the differential equation

$$y^3y' + xy^4 = x$$

[14 marks]

(b) Solve

$$y'' - 2y' = 1.$$

[6 marks]

Question 7

(a) Solve the boundary value problem

$$y'' + 4y' + 4y = 5\sin 2x, \qquad y(0) = 1, \quad y'(0) = 0.$$

[10 marks]

(b) Using the substitution $u = \ln x$. Find the general solution of

$$2x^2y'' - 3xy' + 2y = 0.$$

[10 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$