

# University of Swaziland

## Supplementary Examination, July 2011

### BSc II, Bass II, BEd II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. **Submit solutions to ONLY FIVE questions.**
4. Show all your working.
5. A Table of Laplace Transforms is provided at the end of the question paper.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

### Question 1

- (a) Find the general solution

$$(y^2 - 1)dx - 2(2y + xy)dy = 0.$$

[8 marks]

- (b) Use the method of Laplace transforms to solve

$$\ddot{y}(t) + 2\dot{y}(t) + y(t) = 1, \quad y(0) = 2, \quad \dot{y}(0) = -2.$$

[12 marks]

### Question 2

Solve the following differential equations

- (a)

$$xdy - (y + x^3e^x)dx = 0$$

[8 marks]

- (b)

$$y'' + 4y' + 3y = 5 \sin 2x.$$

[12 marks]

### Question 3

- (a) Show that

$$y = ce^{-\int \frac{q(x)}{p(x)} dx}$$

is a solution to the differential equation

$$p(x)y' + q(x)y = 0$$

where  $c$  is an arbitrary constant.

[6 marks]

- (b) Solve the following differential equation

$$(x + y)dx + (3x + 3y - 4)dy = 0.$$

[14 marks]

**Question 4**

Find the series solution of

$$(x^2 + 1)y'' + xy' - y = 0$$

about  $x = 0$ .

[20 marks]

**Question 5**

Solve the following differential equations

(a)

$$y^{iv} + 5y'' - 36y = 0.$$

[6 marks]

(b)

$$y^{iv} - 10y''' + 25y'' = -4.$$

[9 marks]

(c)

$$x dx + y e^{-x^2} dy = 0.$$

[5 marks]

**Question 6**

(a) Use two methods to solve the differential equation

$$2xy dy + (x^2 - y^2) dx = 0$$

[14 marks]

(b) Solve

$$2x^2 y'' - 3xy' - 3y = 0.$$

[6 marks]

**Question 7**

Solve the following differential equations

(a)

$$y'' + 4y' + 4y = e^{-2x}.$$

[12 marks]

(b)

$$dy - (2xy^2 + 6xy)dx = 0.$$

[8 marks]

## Table of Laplace Transforms

$f(t)$	$F(s)$
$t^n$	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
$e^{at}$	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}(e^{at} - e^{bt})$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}(ae^{at} - be^{bt})$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2 + a^2}$
$\cos(at)$	$\frac{s}{s^2 + a^2}$
$\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2 + a^2)^2}$
$e^{at} \sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$
$e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$
$\sinh(at)$	$\frac{a}{s^2 - a^2}$
$\cosh(at)$	$\frac{s}{s^2 - a^2}$
$\sin(at) \sinh(at)$	$\frac{2a^2}{s^4 + 4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1} f(0) - \dots - f^{(n-1)}(0)$