UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2010/2011

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	THIS PAPER CONSISTS OF SEVEN QUESTIONS.
		ANSWER ANY FIVE QUESTIONS.
		ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find the equation of a straight line passing through the points $(-1, 2)$ and $(2, 4)$. [3 marks]

   (b) Find the centre and radius of the circle with equation

$$x^2 + y^2 - 4x + 5y - 1 = 0$$

[4 marks]

   (c) Use the dot product to find the unit vector perpendicular to both $\vec{a} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{b} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$. [6 marks]

   (d) Find a, b if

$$(a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + \hat{\mathbf{k}}) \times (2\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}) = \hat{\mathbf{i}} - \hat{\mathbf{j}}$$

[7 marks]

QUESTION 2

2. (a) Let

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

   Evaluate $\det(A)$ by

   i. cofactor expansion along the first column of A,

   ii. direct formula for a 3×3 matrix. [3,3 marks]

   (b) Let

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

   Find

   i. All the minors,

   ii. All the cofactors,

   iii. Adjoint matrix,

   iv. Inverse matrix. [2,1,1,2 marks]

   (c) Solve the linear system

$$\begin{aligned} x_1 \qquad\quad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8, \end{aligned}$$

   i. by Cramer's rule,

   ii. by Gaussian elimination. [4,4 marks]

QUESTION 3

3. (a) Apply a derivative to sketch the curve of $y = x + \frac{4}{x}$. [4 marks]

   (b) When a rock falls into a pond it creates a circular ripple, the radius of which is increasing at the rate of $3m/s$. At what rate is the area of the ripple increasing when the radius is $15m$? [5 marks]

   (c) Check Rolle's theorem for the function $f(x) = 1 - \sqrt[3]{x^2}$ on the interval $[-1, 1]$. [5 marks]

   (d) Apply L'Hospital rule to evaluate the following limits.

      i. $\lim_{x \to 0^+} \frac{e^x - 1}{x^2}$.

      ii. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}$. [3,3 marks]

QUESTION 4

4. (a) Express the polynomial

$$p(x) = 7 - 8x + 4x^2 + 5x^3 - 3x^4$$

   in Taylor form about 1. [8 marks]

   (b) i. Use the quadratic approximation formula to compute $\cos 0.2$, and

      ii. estimate the error term. [4,2 marks]

   (c) Consider the function

$$f(x) = x^{\frac{7}{2}}$$

   Construct near the origin the Taylor polynomial of

      i. degree three,

      ii. degree four. [3,3 marks]

QUESTION 5

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5. (a) For the function $f(x,y) = \dfrac{4\sqrt{x}}{y^3}$

   i. find $\dfrac{\partial f}{\partial x}$,

   ii. find $\dfrac{\partial f}{\partial y}$. [3,3 marks]

(b) If $u = \dfrac{x-y}{1+xy}, x = \tan s, y = \tan t$, find u'_s and u'_t at $s = 0, t = \dfrac{\pi}{4}$. [7 marks]

(c) Find and classify the stationary points of the following function.

$$z = \frac{x^2}{2p} + \frac{y^2}{2q},\ p > 0,\ q > 0.$$

[7 marks]

QUESTION 6

6. (a) Apply Lagrange's method to find the extremum of $f(x,y) = xy$ subject to constraint $x + y = 1$. [8 marks]

(b) Find the area of the region enclosed between the curves $y = x^2$ and $y = x + 2$. [6 marks]

(c) The portion of the curve $y = x^2$ between $x = 1$ and $x = 3$ is rotated about the x-axis. Find the volume of the solid of revolution formed. [6 marks]

QUESTION 7

7. (a) Derive the formula for the length of the curve

$$y = f(x),\ a \le x \le b.$$

[5 marks]

(b) Find the area of the surface generated by rotating the curve C:

$$z = \sqrt{3}x,\ 1 \le x \le 2$$

about the z-axis. [6 marks]

(c) Determine the volume of the solid below the surface $z = xy$ and over the region R bounded by the line $y = 3x$, the curve $y = 4 - x^2$ and the y-axis. [9 marks]