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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2010/2011

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M 215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	THIS PAPER CONSISTS OF SEVEN QUESTIONS. ANSWER ANY FIVE QUESTIONS. ONLY NON-PROGRAMMABLE CALCULATORS MAY BE USED.

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Find the equation of a straight line perpendicular to the line $y - 3x = 2$ and passing through the origin. [3 marks]

   (b) Find the centre and radius of the circle with equation

$$2x^2 + 2y^2 + 6x - 7y + 5 = 0$$

[4 marks]

   (c) Use the dot product to find the unit vector perpendicular to both $\vec{a} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}}$ and $\vec{b} = 3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}$. [6 marks]

   (d) Use the cross product to find the vector perpendicular to $\hat{\mathbf{i}} + 2\hat{\mathbf{k}}$ and $\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and hence find the area of the triangle with these two vectors as adjacent sides. [7 marks]

QUESTION 2

2. (a)

$$A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$$

   Evaluate $\det(A)$ by

   i. cofactor expansion along the first row of A,
   ii. direct formula for a 3×3 matrix. [3,3 marks]

   (b) Let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

   Find

   i. All the minors,
   ii. All the cofactors,
   iii. Adjoint matrix,
   iv. Inverse matrix. [2,1,1,2 marks]

   (c) Solve the linear system

$$\begin{aligned} 4x + 5y + \quad &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1, \end{aligned}$$

   i. by Cramer's rule,
   ii. by Gaussian elimination. [4,4 marks]

QUESTION 3

3. (a) Apply a derivative to sketch the curve of $y = 1 - x^2 + x^4$. [4 marks]

   (b) Water is flowing into a conical tank at the rate of 3 cubic meters per minute. The tank has a radius of 2 meters at the top and a depth of 4 meters. How fast is the water level rising when the water is 1 meter deep? [5 marks]

   (c) Apply the Mean Value Theorem to find a point on the curve $y = x^3$ at which a tangent line is parallel to the chord connecting $A(-1,-1)$ and $B(2,8)$. [5 marks]

   (d) Apply L'Hospital rule to evaluate the following limits.

   i. $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x^2 - 5x + 4}$.

   ii. $\lim\limits_{x \to 0} \dfrac{\sin x}{x}$. [3,3 marks]

QUESTION 4

4. (a) Express the polynomial

$$p(x) = 1 + x + x^2 + x^3$$

   in Taylor form about 1. [8 marks]

   (b) i. Use the quadratic approximation formula to compute $e^{-0.1}$, and

   ii. estimate the error term. [4,2 marks]

   (c) Consider the function

$$f(x) = x^{\frac{11}{3}}$$

   Construct near the origin the Taylor polynomial of

   i. degree three,

   ii. degree four. [3,3 marks]

QUESTION 5

5. (a) For the function $f(x,y) = x^2 + xy^2 + y^3$
   i. find $f_x(1,-3)$,
   ii. find $f_y(2,1)$. [3,3 marks]

(b) If $w = u^2 + v^2, u = \dfrac{x+1}{y}, v = \dfrac{y+1}{x}$, find w_x and w_y at $x = 0, y = 2$. [7 marks]

(c) Find and classify the stationary points of the following function.

$$z = \frac{x^2}{2p} - \frac{y^2}{2q},\ p > 0,\ q > 0.$$

[7 marks]

QUESTION 6

6. (a) Apply Lagrange's method to find the extremum of $f(x,y) = x^2 + y^2$ subject to constraint $\dfrac{x}{a} + \dfrac{y}{b} = 1$. [8 marks]

(b) Find the area of the region enclosed between the curves $y = x^3$ and $y = x^2$. [6 marks]

(c) The loop of the curve

$$y^2 = x(x-2)^2,\ 0 \le x \le 2,$$

is rotated about the x-axis. What is the volume of the solid so formed. [6 marks]

QUESTION 7

7. (a) Derive the formula for the arc length of the curve

$$x = \varphi(t), y = \psi(t),\ t_0 \le t \le \tau.$$

[5 marks]

(b) Find the area of the surface generated by rotating the curve C:

$$x = \frac{1}{4}y^2,\ 0 \le y \le 2$$

about the x-axis. [6 marks]

(c) Determine the volume of the solid below the surface $z = x^2 + 2y$ and over the region

$$R = \{(x,y) : 0 \le x \le 2, x^2 \le y \le 2x\}.$$

[9 marks]