UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2010/2011

B.Sc. / B.Ed. / B.A.S.S. II

TITLE OF PAPER	:	DYNAMICS I
COURSE NUMBER	:	M255
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Find the projection of the vector $\mathbf{b} = (-4, 2, 2)$ onto the vector $\mathbf{a} = (3, 0, 4)$. [6]

(b) Prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and has half its length. [6]

(c) Find the equation of the plane through the points $P = (0, 0, 1)$, $Q = (2, 1, 0)$ and $R = (1, 1, 1)$. [8]

QUESTION 2

(a) At time $t = 0$ a particle of mass m is located at $z = 0$ and is traveling vertically downwards with speed v_0. If the resisting force is $-\beta v$, where v is the speed at time t, find

(i) the speed at any time t,

(ii) the distance traveled after time t, and

(iii) the acceleration at any time t. [6,4,2]

(b) A particle is projected vertically upwards with initial speed u. Gravity acts, as does air resistance, which is given by kv per unit mass, where k is a constant and v is the speed of the particle. Find the time taken to reach the maximum height. [8]

QUESTION 3

parametrization of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1; \qquad a, b > 0,$$

traversed in the counterclockwise direction is given by $x = \cos t$, $y = \sin t$; $t \geq 0$. Suppose that a particle moves along this ellipse in the counterclockwise direction. Find:

(a) The position vector $\mathbf{r}$; [1]

(b) the velocity vector $\mathbf{v}$; [1]

(c) the speed $|\mathbf{v}|$; [1]

(d) the acceleration vector $\mathbf{a}$; [1]

(e) the magnitude of the acceleration $|\mathbf{a}|$; [1]

(f) the unit tangent vector $\hat{\mathbf{T}}$; [2]

(g) the principal unit normal vector $\hat{\mathbf{N}}$; [5]

(h) the curvature κ; [2]

(i) the unit binormal vector $\hat{\mathbf{B}}$; and [2]

(j) the tangential and normal components of the acceleration of the particle at the point $P\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$. [4]

QUESTION 4

(a) In cylindrical coordinates (s, θ, z), the position vector of an arbitrary point (x, y, z) is given by

$$\mathbf{r}(s, \theta, z) = s\cos\theta\hat{\mathbf{i}} + s\sin\theta\hat{\mathbf{j}} + z\hat{\mathbf{k}}.$$

Find:

(i) $\hat{s}$; [2]

(ii) $\hat{\theta}$; [2]

(iii) $\hat{z}$; [2]

(iv) the velocity vector $\mathbf{v}$; [2]

(v) $\dot{\hat{s}}$; [2]

(vi) $\dot{\hat{\theta}}$; [2]

(vii) $\dot{\hat{z}}$; and [1]

(viii) the acceleration vector [2]

for any particle moving in this coordinate system.

(b) Prove that if $\mathbf{v}$ is any vector of constant length, then $\mathbf{v}$ and $\frac{d\mathbf{v}}{dt}$ are orthogonal. [3]

(c) If $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$, prove that $\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$. [2]

QUESTION 5

(a) A particle is projected with velocity $\mathbf{u}$ from a point O in a vertical plane through the line of greatest slope of a plane inclined at an angle β to the horizontal. After time T, the particle strikes the inclined plane at the point P, at a distance R from O. If $\mathbf{u}$ makes an angle α with the horizontal, and if $|\mathbf{u}| = u$, show that:

(i) $T = \dfrac{2u\sin(\alpha - \beta)}{g\cos\beta}$ and $R = \dfrac{u^2[sin(2\alpha - \beta) - \sin\beta]}{g\cos^2\beta}$;

(ii) for constant u and β, R is maximum when $\alpha = \dfrac{\pi}{4} + \dfrac{\beta}{2}$. [9,3]

(b) Under the influence of a central force field, a particle moves in a circular orbit through the origin. Find the law of force. [8]

QUESTION 6

(a) An inductor of 2 henries, a resistor of 4 ohms, and a capacitor of 0.05 farads are connected in series with a battery of $E = 100$ volts. At $t \leq 0$ the charge on the capacitor and the current in the circuit are zero. Find the charge and current at any time $t > 0$. [8]

(b) Solve the problem in (a) if now the battery is of e.m.f. $E = 100\sin(4t)$. [12]

QUESTION 7

(a) A particle moves on the x axis, attracted to the origin O by a force proportional to its distance from O. If the particle starts from rest at $x = 5$ cm and reaches $x = 2.5$ cm for the first time after 2 seconds, find:

(i) the position at any time t after it starts;

(ii) the magnitude of the velocity at $x = 0$;

(iii) the amplitude, period, and frequency of the vibration; and

(iv) the acceleration. [6,3,2,1]

(b) (i) A 7 kg weight suspended at the end of a vertical spring stretches it 5 cm. Assuming that a damping force numerically equal to 0.2 times the velocity is acting on the system, find the position of the weight at any time t if initially the weight is pulled down 10 cm and released.

(ii) Is the motion in (i) oscillatory, overdamped, or critically damped? [7,1]

END OF EXAMINATION