UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2010/2011

B.Sc. / B.Ed. / B.A.S.S.III

TITLE OF PAPER	:	DYNAMICS II
COURSE NUMBER	:	M355
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Give the definitions and some examples of

(i) non-holonomic constraints,

(ii) scleronomic systems. [3,3]

(b) Prove that a mass connected to a spring forms a conservative system. [3]

(c) Prove the Cancellation of Dot Property Lemma

$$\frac{\partial \dot{\bar{r}}_\nu}{\partial \dot{q}_i} = \frac{\partial \bar{r}_\nu}{\partial q_i}.$$

[7]

(d) Consider a mathematical pendulum.

(i) Derive Lagrange's equation,

(ii) Solve it for small angle. [2,2]

QUESTION 2

(a) The Lagrangian for a certain dynamical system is given by

$$L = \frac{1}{2}\left(\dot{x}^2 + \dot{y}^2\right) + \frac{\omega}{2}\left(-\dot{x}y + \dot{y}x\right),$$

where ω is a constant. Write down Lagrange's equations. [6]

(b) Prove that for holomonic, scleronomic systems,

$$\sum_{i=1}^{n} \dot{q}_i \frac{\partial T}{\partial \dot{q}_i} = 2T,$$

in the usual notations. [8]

(c) Masses m_1 and m_2 are connected by an inextensible string of negligible mass, which passes over a smooth peg. Mass m_1 is located on a smooth inclined plane of angle α, and mass m_2 is hanging. Find the acceleration of m_1. [6]

QUESTION 3

(a) Derive Hamilton's equations if $H = H(q, p, t)$. [6]

(b) For the mass m attached to the horizontal spring of stiffness c, find,

(i) generalized momenta,

(ii) Hamiltonian,

(iii) Hamilton's equations. [2,2,2]

(c) Consider a transformation for the mathematical pendulum $x = l \sin\phi$.

(i) Show that $p_x = \dfrac{p_\phi}{l\cos\phi}$,

(ii) Prove that the transformation $(\phi, p_\phi) \longrightarrow (x, p_x)$ is canonical. [5,3]

QUESTION 4

(a) Prove that generalized momenta conjugate to a cyclic coordinate is conserved.[6]

(b) Consider a system with two degrees of freedom with kinetic and potential energy as follows;

$$T = \frac{M+m}{2}\dot{x}^2 + m\dot{x}\dot{y}\cos\alpha + \frac{m}{2}\dot{y}^2,$$

$$\Pi = -mgy\sin\alpha,$$

where M, m, g, α are the constants. Find

(i) generalized momenta p_x and p_y,

(ii) Hamiltonian. [4,10]

QUESTION 5

(a) Consider the physical quantities

$$\mathscr{U}(q,p), \quad \mathscr{V}(q,p), \quad \text{and,} \quad \mathscr{W}(q,p).$$

Prove that

$$[\mathscr{U}, \mathscr{V} + \mathscr{W}] = [\mathscr{U}, \mathscr{V}] + [\mathscr{U}, \mathscr{W}].$$

[6]

(b) Let $H = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{2}\omega^2(q_1^2 + q_2^2)$, where ω is a constant. Show that $q_1p_2 - q_2p_1$ is a constant of motion. [6]

(c) (i) Write down, and

(ii) derive Hamilton's equations in Poisson formulation. [2,6]

QUESTION 6

(a) (i) State, and

(ii) prove the Main Lemma of the Calculus of Variations. [1,5]

(b) Find the extremals for the functional

$$\mathscr{U}[y(x)] = \int_0^{\frac{\pi}{2}} [(y')^2 - y^2]\,\mathrm{d}x, \quad y(0) = 0, \quad y\left(\frac{\pi}{2}\right) = 1.$$

[6]

(c) Consider a functional

$$\mathscr{U}[y(x)] = \int_{x_0}^{x_1} F(x, y')\,\mathrm{d}x.$$

(i) What is the first integral, and

(ii) thus find the extremals of

$$\mathscr{U}[y(x)] = \int_{x_0}^{x_1} y'(1 + x^2 y')\,\mathrm{d}x.$$

[2,6]

QUESTION 7

(a) Find extremals for the following functionals

(i)

$$\mathscr{U}[y(x), z(x)] = \int_0^1 (y'^2 + z'^2 + y'z')\,\mathrm{d}x,$$

where $y(0) = 0$, $z(0) = 0$, $y(1) = 1$, and $z(1)$ is free;

(ii)

$$\mathscr{U}[y(x)] = \int_{x_0}^{x_1} (2xy + y'''^2)\,\mathrm{d}x.$$

[8,6]

(b) Find Ostrogradski's equation for the following functional

$$\mathscr{U}[z(x,y)] = \iint_D \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2\right] \mathrm{d}x\,\mathrm{d}y,$$

where $z(x, y)$ is known on the boundary of region D. [6]

END OF EXAMINATION