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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2010/2011

BSc. IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M 455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS. 3. NON PROGRAMMABLE CALCULATORS MAY BE USED.
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial \psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\hat{\theta} + \frac{\partial \psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla \cdot \underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla \times \underline{v} = \frac{1}{r}\det\begin{bmatrix} \hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla \cdot \underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial \lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial \theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v} \cdot \nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v} \times \underline{\omega}$$

$$\nabla \times (\nabla \times \underline{a}) = \nabla\nabla \cdot \underline{a} - \nabla^2\underline{a}$$

QUESTION 1

1. (a) For the velocity field

$$\underline{v} = ax\hat{i} - by\hat{j}$$

where a and b are positive constants, determine

   i. whether the flow field is one-dimensional, two-dimensional or three-dimensional, and why; [1 marks]

   ii. whether the flow is steady or unsteady, and why; [1 marks]

   iii. equation for the streamline through point $(x, y) = (1, 4)$; [5 marks]

   iv. particle path for particle located at (x_0, y_0) at $t = 0$. [6 marks]

(b) Define the air density at a point. [3 marks]

(c) Describe the Eulerarian method of treating motion of a continuous medium. [4 marks]

QUESTION 2

2. (a) Derive the mass conservation equation in general form. [8 marks]

(b) Which of the following sets of equations represent possible incompressible flow cases? Explain.

   i. $u = x + y + z^2$, $v = x - y + z$, $w = 2xy + y^2 + 4$. [2 marks]

   ii. $u = xyzt$, $v = -xyzt^2$, $w = \frac{z^2}{2}(xt^2 - yt)$. [2 marks]

   iii. $u = y^2 + 2xz$, $v = -2yz + x^2yz$, $w = \frac{1}{2}x^2z^2 + x^3y^4$. [2 marks]

(c) Determine the family of stream functions that will yield the velocity field

$$\underline{v} = (x^2 - y^2)\hat{i} - 2xy\hat{j}$$

[6 marks]

QUESTION 3

3. (a) Prove

$$\underline{v} = \text{grad}\,\psi \times \hat{k}$$

in the usual notation, and thus [5 marks]

(b) show that

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = -\frac{\partial \psi}{\partial r}$$

[5 marks]

(c) Incompressible flow around a circular cylinder of radius a is represented by the stream function

$$\psi(r, \theta) = -Ur\sin\theta + \frac{Ua^2 \sin\theta}{r},$$

where U represents the free stream velocity.

i. Obtain an expression for the velocity field. [3 marks]

ii. Find v_r along the circle $r = a$. [2 marks]

iii. Locate the points along $r = a$ where $|\underline{v}| = U$. [2 marks]

(d) Find stream function for the line vortex of circulation k. [3 marks]

QUESTION 4

4. (a) Consider the Rankine's vortex

$$\underline{\omega} = \begin{cases} \Omega\hat{k} & \text{for } r < a \\ \underline{0} & \text{for } r > a \end{cases}$$

i. Find the stream function. [7 marks]

ii. Find velocity v_θ. [5 marks]

(b) Consider the flow field represented by the stream function

$$\psi(x, y) = 10xy + 17$$

i. Is this a possible two-dimensional incompressible flow? [4 marks]

ii. Is the flow irrotational? [4 marks]

QUESTION 5

5. (a) Describe the stress components σ_{xx} and τ_{xy}. [4 marks]

(b) Define Newtonian fluid. [2 marks]

(c) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = u_{\max}\left[1 - \left(\frac{2y}{h}\right)^2\right],$$

where h is the distance separating the plates, and the origin is placed mid-way between the plates. Consider $\mu = 1.1 \times 10^{-3} kg/ms$, $u_{\max} = 0.05 m/s$, $h = 5mm$. Calculate

i. the shear stress on the lower plate and give direction, [4 marks]

ii. the force on a $0.3m^2$ section of the lower plate and give its direction. [2 marks]

(d) A velocity field in a fluid with density of $1500 kg/m^3$ is given by

$$\underline{v} = (Ax - By)t\hat{i} - (Ay + Bx)t\hat{j},$$

where $A = 1s^{-2}$, $B = 2s^{-2}$, x and y are in meters, and t in seconds. Body and viscous forces are negligible.

i. Find the acceleration of a fluid particle at point $(x, y) = (1, 2)$. [6 marks]

ii. Find the pressure gradient at the same point. [2 marks]

QUESTION 6

6. (a) Consider stationary viscous incompressible flow between two stationary plates located at $y = 0$ and $y = 1$. Given that pressure at $x = 0$ and $x = L$ is P_0 and P_L respectively, with $P_0 > P_L$. The effect of body forces is negligible.

i. Write the Navier-Stokes equations for incompressible flow. [2 marks]

ii. Put $\underline{v} = u(x, y)\hat{\imath}$ and simplify the Navier-Stokes equations to show that

$$\frac{\partial P}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2}$$

[5 marks]

iii. Show that

$$P(x) = P_0 - \frac{P_0 - P_L}{L} x$$

[3 marks]

iv. Show that

$$u(y) = y(1 - y)\frac{P_0 - P_L}{2\mu L}$$

[3 marks]

(b) i. Re-write the Navier-Stokes equations in dimensionless form introducing the characteristic length and velocity. [5 marks]

ii. Define Reynolds number. [1 marks]

iii. Find dimension of Reynolds number. [1 marks]

QUESTION 7

7. (a) Consider steady incompressible inviscid potential flow.

i. Show that

$$\underline{v} \times \underline{\omega} = \text{grad}\left[\frac{1}{2}v^2 + \Phi + \frac{p}{\rho}\right]$$

[5 marks]

ii. Derive Bernoulli equation. [5 marks]

(b) Water flows in a circular pipe. At one section the diameter is $0.3m$, the static pressure is $260kpa$ (gage), the velocity is $3m/s$, and the elevation is $10m$ above ground level. At a section downstream at ground level the pipe diameter is $0.15m$. Find the gage pressure at the downstream section, if friction effects may be neglected. [10 marks]