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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION 2010/2011

BSc. IV

TITLE OF PAPER : FLUID DYNAMICS

COURSE NUMBER : M 455

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS :
1. THIS PAPER CONSISTS OF SEVEN QUESTIONS.
2. ANSWER ANY FIVE QUESTIONS.
3. NON PROGRAMMABLE CALCULATORS MAY BE USED.

SPECIAL REQUIREMENTS : NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial\psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} + \frac{\partial\psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla\cdot\underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial\theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla\times\underline{v} = \frac{1}{r}\det\begin{bmatrix}\hat{r} & r\hat{\theta} & \hat{k}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial z}\\ v_r & rv_\theta & v_z\end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla\cdot\underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial\lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial\theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial\theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v}\cdot\nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v}\times\underline{\omega}$$

$$\nabla\times(\nabla\times\underline{a}) = \nabla\nabla\cdot\underline{a} - \nabla^2\underline{a}$$

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QUESTION 1

1. (a) For the velocity field

$$\underline{v} = -ax\hat{\imath} + by\hat{\jmath}$$

where a and b are positive constants, determine

i. whether the flow field is one-dimensional, two-dimensional or three-dimensional, and why; [1 marks]

ii. whether the flow is steady or unsteady, and why; [1 marks]

iii. equation for the streamline through point $(x, y) = (1, 1)$; [5 marks]

iv. parameter equation for particle path located at $(x, y) = (2, 1)$ at $t = 0$. Put $a = b = 2s^{-1}$. [6 marks]

(b) Define the air density at a point. [3 marks]

(c) Describe the Eulerarian method of treating motion of a continuous medium. [4 marks]

QUESTION 2

2. (a) Derive the formula for convective derivative of the density. [6 marks]

(b) Which of the following sets of equations represent possible incompressible flow cases? Explain.

i. $u = 2x^2 + y^2$, $v = x^3 - x(y^2 - 2y)$.

ii. $u = 2xy - x^2 + y$, $v = 2xy - y^2 + x^2$.

iii. $u = xt + 2y$, $v = xt^2 - yt$. [4 marks]

(c) The x component of velocity in steady, incompressible flow field in the xy plane is $u = \dfrac{A}{x}$, where $A = 2m^2/s$, and x is measured in meters. Find the simplest y component of velocity for this flow field. [4 marks]

(d) A uniform flow field $\underline{v}$ is inclined at angle α above the x axis.

i. Evaluate the velocity components u and v. [2 marks]

ii. Determine the stream function for this flow field. [4 marks]

QUESTION 3

3. (a) Show that

$$\underline{v} = \text{grad}\,\psi \times \hat{k}$$

in the usual notation, and thus [5 marks]

(b) prove that

$$v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta}, \quad v_\theta = -\frac{\partial \psi}{\partial r}$$

[5 marks]

(c) The stream function for a certain incompressible flow field is given by the expression

$$\psi(r, \theta) = -Ur\sin\theta + \frac{q\theta}{2\pi},$$

where U represents the free stream velocity.

i. Obtain an expression for the velocity field. [3 marks]

ii. Find the stagnation point(s), [2 marks]

iii. and show that $\psi = 0$ there. [2 marks]

(d) Evaluate the circulation of a line vortex. [3 marks]

QUESTION 4

4. (a) The vorticity of a certain incompressible flow is given by the following formula

$$\underline{w} = \begin{cases} -Ar\sin\theta & \text{for } r < a \\ 0 & \text{for } r > a \end{cases}$$

Find the corresponding stream function. [12 marks]

(b) A flow is represented by the velocity field

$$\underline{v} = 10x\hat{\imath} - 10y\hat{\jmath} + 30\hat{k}$$

Determine if the field is

i. a possible incompressible flow, [4 marks]

ii. irrotational flow. [4 marks]

QUESTION 5

5. (a) Explain the stress components σ_{xx} and τ_{xy}. [4 marks]

(b) Define the Newtonian fluid. [2 marks]

(c) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = u_{\max}\left[1 - \left(\frac{2y}{h}\right)^2\right],$$

where h is the distance separating the plates, and the origin is placed mid-way between the plates. Consider $\mu = 1.1 \times 10^{-3} kg/ms$, $u_{\max} = 0.3 m/s$, $h = 0.5 mm$. Calculate

i. the shear stress on the lower plate and give direction, [4 marks]

ii. the force on a $0.5 m^2$ section of the upper plate and give its direction. [2 marks]

(d) An incompressible flow field is given by

$$\underline{v} = (Ax + By)t\hat{i} - Ay\hat{j},$$

where $A = 1s^{-2}$, $B = 2s^{-2}$, the coordinates are in meters.

i. Find the acceleration of a fluid particle at point $(x, y) = (1, 2)$. [6 marks]

ii. Find the pressure gradient at the same point, if $\underline{g} = -g\hat{j}$ and fluid is water, $\rho = 1000 kg/m^3$. [2 marks]

QUESTION 6

6. (a) The plane $y = 0$ oscilates so that its velocity is in the plane $y = 0$ and has magnitude $v \cos \omega t$, where v and ω are constants. Above the plane there is viscous incompressible fluid. Body forces are negligible.

   i. Write the Navier-Stokes equations for incompressible flow. [2 marks]

   ii. Show that
$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2}$$
[5 marks]

   iii. Separate variables to how that
$$U(y,t) = Re\left\{ v \exp\left(i\omega t - \sqrt{\frac{i\omega}{\nu}} y \right) \right\}$$
[5 marks]

   iv. Derive the mass conservation equation for incompressible flow in dimensionless form. [4 marks]

   v. A. Define similar flows, and [2 marks]

   B. Explain how the idea of similarity is used to design the experimental models. [2 marks]

QUESTION 7

7. (a) For steady incompressible inviscid potential flow

   i. Prove the following formula
$$\underline{v} \times \underline{\omega} = \text{grad}\left[\frac{1}{2}v^2 + \Phi + \frac{p}{\rho}\right]$$
[5 marks]

   ii. Derive Bernoulli equation. [5 marks]

   (b) Water flows steadily up a vertical $0.1m$ diameter pipe and out the nozzle, which is $0.05m$ in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be $20m/s$. Calculate the gage pressure required at inlet, assuming frictionless flow. [10 marks]