# University of Swaziland

## Final Examination, December 2011

## BSc I, EEng I, BEd I

**Title of Paper** : Algebra, Trig. and Analytic Geometry

**Course Number** : M111

**Time Allowed** : Three (3) hours

**Instructions** :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

## Question 1

(a) Consider the AP

$$97,\ 94,\ 91,\ 88, \cdots .$$

i. Find the formula for the

A. $n$th term [2 marks]

B. sum of the first $n$ terms [3 marks]

ii. If the sum of the first $n$ terms is 160, find $n$. [5 marks]

(b) Expand $\left(1-i\sqrt{3}\right)^6$ and leave your answer in the form $a+ib$, using the

i. binomial theorem [5]

ii. de Moivre's theorem [5]

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## Question 2

(a) Describe the locus of points represented by the given equation.

$$y^2+4y+20x-6=0. \qquad [6 \text{ marks}]$$

Make a sketch of the curve.

(b) Evaluate

$$\begin{vmatrix} 3 & 2 & 0 & -3 \\ -2 & 0 & 4 & 5 \\ 2 & -6 & -4 & 0 \\ 0 & 1 & 0 & 3 \end{vmatrix} \qquad [6]$$

(c) Prove

$$\cos(A+B)\cos(A-B)=\cos^2 A-\sin^2 B. \qquad [6]$$

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## Question 3

(a) Find the middle term of the binomial expansion of

$$\left(\frac{3x}{y^2} - \frac{y}{2\sqrt{x}}\right)^{16}. \qquad [6]$$

(b) Find the value of

$$27\sum_{n=0}^{\infty}\left(\frac{2}{3}\right)^n. \qquad [4]$$

(c) The remainder when

$$P(x) = 2x^4 + 5x^3 + Ax^2 - 5x + 3,$$

is divided by $x - 2$ is 45.

i. Find the value of $A$. [2]

ii. Factorise $P(x)$ and hence find all its roots. [8]

## Question 4

(a) Solve for $x$ :

i. $e^{\ln(2x^2-1)} = 7$ [4]

ii. $\log_2 x = 3 - \log_2(x+2)$ [6]

(b) Use mathematical induction to prove

$$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1},$$

where $n \in \mathbb{Z}$ and $n \geqslant 1$. [10]

## Question 5

(a) Use synthetic division to divide

$$\frac{x^4 - 2x^3 + 4x - 9}{x + 2}. \qquad [5]$$

(b) Use Cramer's rule to solve

$$\begin{aligned} x + y - z &= 6, \\ x - 2y + z &= -3, \\ 2x + y - 3z &= 14. \end{aligned}$$

[15]

## Question 6

(a) Divide

$$\frac{x^5 + x^4 - 5x^2 - 8x + 7}{x^2 + 3}. \qquad [7]$$

(b) Find the the first 4 terms of the binomial expansion of

$$\left(\frac{1}{a} + ab\right)^{-2}. \qquad [7]$$

(c) Find all the fourth roots of $-81$. [6]

**Question 7**

(a) Given that $z = 1 - 2i$ is a root of

$$P(z) = z^3 + z + 10,$$

find the two other roots. [5]

(b) Solve for $x$ (in the range $0 \leqslant x < 2\pi$)

$$\cos 2x + \sin x = 0. \quad [8]$$

(c) Use mathematical induction to prove

$$\cos(\theta + n\pi) = (-1)^n \cos\theta,$$

where $n \in \mathbb{Z}$. [7]