
University of Swaziland



Supplementary Examination, July 2012

BSc I, EEng I, BEd I, BASS I

Title of Paper : Introduction to Calculus

Course Number : M115

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate

(i) $\lim_{x \rightarrow -1} \frac{x+1}{x^2-1}$ [3 marks]

(ii) $\lim_{x \rightarrow \infty} \frac{2x+1}{x^2+x-3}$ [3 marks]

(b) Differentiate and simplify

(i) $y = x\sqrt{2x-1}$ [4 marks]

(ii) $y = \tan^{-1}\left(\frac{x}{2}\right)$ [3 marks]

(c) Show that $y = 5 \sin 3t - 4 \cos 3t$ satisfies the differential equation

$$\ddot{y} + 9y = 0, \quad [7 \text{ marks}]$$

where $\dot{y} = \frac{dy}{dt}$.

Question 2

(a) Differentiate

(i) $y = x^x$ [5 marks]

(ii) $F(x) = \ln(\sin 2x)$ [5 marks]

(b) Integrate

$$\int \frac{x}{x^2 - 2x - 8} dx. \quad [10 \text{ marks}]$$

Question 3

(a) Find $\frac{df}{dx}$ using the *limit definition*, for

$$f(x) = \frac{1}{\sqrt{x}}. \quad [8 \text{ marks}]$$

(b) Integrate

i. $\int \frac{3x^2}{\sqrt{x^3 + 5}} dx$ [6 marks]

ii. $\int_0^{\frac{1}{2}\pi} \sin^2 \theta \cos \theta d\theta$ [6 marks]

Question 4

(a) Evaluate

i. $\lim_{\theta \rightarrow 0} \frac{\tan 2\theta}{\theta}$ [4 marks]

ii. $\lim_{\theta \rightarrow 0} \frac{e^\theta - e^{-\theta}}{e^\theta + e^{-\theta}}$ [3 marks]

(b) Use Leibnitz rule to find $\frac{d^4 f}{dx^4}$, given

$$f(x) = x^5 \ln x. \quad [7 \text{ marks}]$$

(b) The base of a rectangle rests on the x -axis while the two top vertices are on the parabola $y = 16 - \frac{1}{3}x^2$. Find the largest such rectangle. [6 marks]

Question 5(a) Find y' .

i. $y = e^2 + \ln(x) - 4x^3 - 8x^{\frac{3}{4}} + \frac{8}{x}$ [3 marks]

ii. $y = \frac{2x - 1}{4x + 3}$ [6 marks]

(b) Find the equation of the normal to the curve $x^2 - xy^2 + 8 = 0$ at the point $(1, -3)$. [4 marks]

(c) Integrate

$$\int x^2 e^{2x} dx.$$
 [7 marks]

Question 6

(a) Make a sketch of the graph of

$$y = H(x + 2) - H(x - 5)$$

where $H(x)$ denotes the Heaviside step function. [5 marks]

(b) Given

$$y = \tan 2x,$$

find y''' . [8 marks]

(c) Integrate

$$\int \frac{dx}{x^2 - 4x + 5}.$$
 [7 marks]

Question 7

(a) Differentiate

$$\Gamma(\theta) = \sin^4 \theta - \cos^4 \theta$$

and show that $\frac{d\Gamma}{d\theta} = 2 \sin 2\theta$. [6 marks]

(b) Integrate

i. $\int \frac{dx}{\sqrt{9-x^2}}$ [7 marks]

ii. $\int_0^4 x\sqrt{1-x} dx$ [7 marks]
