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University of Swaziland

Supplementary Examination, 2011/2012

BSc II, Bass II, BEd II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

QUESTION 1

1.1 Find the absolute maximum and absolute minimum values of the function

$$f(x) = \frac{x}{x^2+1} + 1$$

on the interval $[0, 2]$. [6

1.2 Consider the function $f(x) = x^4 - 2x^2 + 3$.

1.2.1 Find the intervals on which f is increasing or decreasing. [4

1.2.2 Find the local maximum and local minimum values of f. [3

1.2.3 Find the intervals on which f is concave up and concave down. [4

1.2.4 Find the inflection points of f. [3

QUESTION 2

Use L'Hopital's Rule to evaluate the following limits.

2.1 $\lim\limits_{x\to 0} \dfrac{e^x - e^{-x} - 2x}{x - \sin x}$ [4

2.2 $\lim\limits_{x\to\infty} xe^{1/x}$ [5

2.3 $\lim\limits_{x\to 1} \left(\dfrac{1}{x-1} - \dfrac{1}{\ln x}\right)$ [6

2.4 $\lim\limits_{x\to\infty} x^{2/x}$ [5

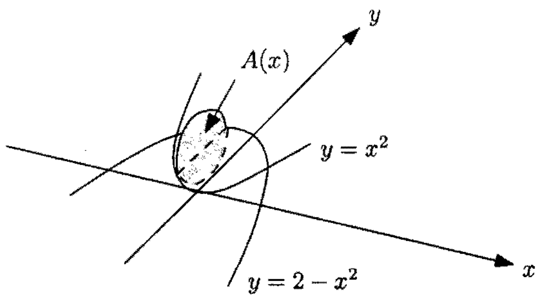
QUESTION 3

3.1 1200 cm^2 of material is available to make a box with a square base and an open top. Find the largest possible volume of the box. [1

3.2 Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$. [1

QUESTION 4

4.1 Find the volume of the solid that lies between the planes $x = -1$ and $x = 1$ and whose cross-sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$ (see below).



[1

4.2 Use cylindrical shells to find the volume of the solid obtained when the region bounded by the curve $y = 3x - x^2$ and the x-axis is rotated about the vertical line $x = -1$. [1

QUESTION 5

5.1 Find the length of the curve with parametric equations $x = \cos^3 t$, $y = \sin^3 t$, $0 \leq t \leq \pi/2$. [***Hint:*** $1 + \cos t = 2\cos^2\left(\frac{t}{2}\right)$.] [1

5.2 The line segment $x = 1 - y$, $0 \leq y \leq 1$ is rotated about the y-axis to generate an open cone. Find its surface area. [1

QUESTION 6

6.1 Investigate the convergence of each series.

6.1.1 $\displaystyle\sum_{n=0}^{\infty} \frac{2^n + 5}{3^n}$ 6.1.2 $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{1+n}\right)^n$ [5

6.2 Find the radius of convergence and interval of convergence of the series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{\sqrt{n}}$. [1

QUESTION 7

7.1 Determine whether the sequence whose nth term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ is convergent or not. If it is convergent, find $\lim_{n\to\infty} a_n$. [1

7.2 Consider the sequence $\{a_n\}$ defined recursively by

$$a_1 = 2 \qquad a_{n+1} = \tfrac{1}{2}(a_n + 6) \quad \text{for } n = 1, 2, 3, \ldots$$

7.2.1 Use mathematical induction to show that $a_{n+1} > a_n$ for all $n \geq 1$. [4

7.2.2 Use mathematical induction to show that $a_n < 6$ for all n. [4

7.2.3 Use your answers to 7.2.1 and 7.2.2 to determine whether or not the sequence is convergent. [2

END OF EXAMINATION PAPER