University of Swaziland

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Final Examination, May 2012

B.Sc II, B.A.S.S II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. **Submit solutions to ONLY FIVE questions.**
4. Show all your working.
5. A Table of Laplace Transforms is provided at the end of the question paper.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Prove that if $f(x)$ and $g(x)$ are two different solutions of

$$\frac{dy}{dx} + P(x)y = Q(x)$$

then $c(f - g) + g$ is also a solution of the differential equation where c is a constant. [3]

(b) Solve the following differential equations

(i) $(x+2)\frac{dy}{dx} + y = f(x)$

where $y(0) = 4$ and $f(x) = \begin{cases} 2x, & 0 \leq x < 2; \\ 4, & x \geq 2. \end{cases}$ [6]

(ii) $\left(x \tan \frac{y}{x} + y\right)dx - xdy = 0$ [7]

(c) Show that $5x^2y^2 - 2x^3y^2 = 1$ is an implicit solution of the differential equation

$$x\frac{dy}{dx} + y = x^3y^3.$$

[4]

Question 2

(a) Consider a differential equation of the form

$$(y + xf(x^2 + y^2))dx + (yf(x^2 + y^2) - x)dy = 0$$

(i) Show that an equation of this form is not exact. [2]

(ii) Show that $\frac{1}{x^2 + y^2}$ is an integrating factor of an equation of this form. [6]

(b) Solve the following differential equation using two methods

$$(2x^2 + 2xy + y^2)dx + (x^2 + 2xy)dy = 0.$$

[12]

Question 3

(a) Prove that if

$$M(x, y)dx + N(x, y)dy = 0$$

is a homogeneous equation, then the change of variables $x = uy$ transforms this equation into a separable equation with variables u and x. [6]

(b) Solve the following differential equation

$$(4x + 3y + 1)dx + (x + y + 1)dy = 0.$$

[14 marks]

Question 4

(a) Given that the following differential equation

$$\mu(x)(P(x)y - Q(x))dx + \mu(x)dy = 0$$

is exact. Find $\mu(x) \neq 0$, given $P(x)$ and $Q(x)$ are continuous functions. [5]

(b) Given $y = x$ is a solution of

$$(x^2 + 1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

find another linearly independent solution by reducing the order. [15]

Question 5

Solve the following differential equations

(a) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = e^{3x}.$ [6]

(b) $\dfrac{dy}{dx} + 2xy = -xy^4.$ [6]

(c) $\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} + 6y = 0.$ [3]

(d) $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} - \dfrac{4}{x^2}y = 0.$ [5]

Question 6

(a) Solve the following differential equation

$$y'' + y = \sec x.$$

Use the method of variation of parameters to find the particular solution. [10]

(b) Solve using the method of Laplace transforms

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 8y = 0, \qquad y(0) = 3, \quad y'(0) = 6.$$

[10]

Question 7

Find the series solution of the differential equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (2x^2 + 1)y = 0$$

about $x = 0$. [20 marks]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$