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University of Swaziland

Supplementary Examination, July 2012

B.Sc II, B.A.S.S. II, B.Ed II, B.Eng II

Title of Paper : Ordinary Differential Equations

Course Number : M213

Time Allowed : Three (3) Hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions. **Submit solutions to ONLY FIVE questions.**
4. Show all your working.
5. A Table of Laplace Transforms is provided at the end of the question paper.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) The differential equation

$$y''' + 2y'' - y' - 2y = e^x + x^2$$

has

$$y_c = c_1 e^x + c_2 e^{-x} + c_3 e^{2x}$$

as the complementary solution. Find the particular solution for the differential equation. [7]

(b) Solve using Laplace transforms

(i) $\dot{y}(t) - 5y(t) = e^{5t}, \quad y(0) = 0.$ [5]

(ii) $\ddot{y}(t) + 16y(t) = 2\sin 4t, \quad y(0) = -\frac{1}{2}, \ \dot{y}(0) = 0.$ [8]

Question 2

(a) Solve the boundary value problem

$$y'' + 4y' + 4y = 5\sin 2x, \quad y(0) = 1, \ y'(0) = 0.$$

[10]

(b) Using the substitution $u = \ln x$. Find the general solution of

$$2x^2y'' - 3xy' + 2y = 0.$$

[10]

Question 3

Find the series solution of

$$(x^2 + 1)y'' + xy' - y = 0$$

about $x = 0$. [20]

Question 4

(a) Show that

$$y = ce^{-\int \frac{q(x)}{p(x)} dx}$$

is a solution to the differential equation

$$p(x)y' + q(x)y = 0$$

where c is an arbitrary constant. [6]

(b) Solve the following differential equation

$$(x + y)dx + (3x + 3y - 4)dy = 0.$$

[14]

Question 5

Find the general solution of the following differential equations

(a)

$$y' = \frac{2y^4 + x^4}{xy^3}$$

[10]

(b)

$$xydx + (1 + x^2)dy = 0$$

[10]

Question 6

(a) Solve

$$y' = \frac{2 + ye^{xy}}{2y - xe^{xy}}$$

[8]

(b) Show that the solution for the linear differential equation

$$p(x)y'(x) + q(x)y(x) = r(x)$$

is given by

$$y(x) = e^{-\int \frac{q(x)}{p(x)}dx}\left(\int \frac{r(x)}{p(x)}e^{\int \frac{q(x)}{p(x)}dx}dx + c\right).$$

Hence solve

$$xy' + 2y = 4x^2.$$

[12]

Question 7

Solve the following differential equations

(a)

$$y^{iv} + 5y'' - 36y = 0.$$

[6]

(b)

$$y^{iv} - 10y''' + 25y'' = -4.$$

[9]

(c)

$$xdx + ye^{-x^2}dy = 0.$$

[5]

Table of Laplace Transforms

$f(t)$	$F(s)$
t^n	$\frac{n!}{s^{n+1}}$
$\frac{1}{\sqrt{t}}$	$\sqrt{\frac{\pi}{s}}$
e^{at}	$\frac{1}{s-a}$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$\frac{1}{a-b}\left(e^{at}-e^{bt}\right)$	$\frac{1}{(s-a)(s-b)}$
$\frac{1}{a-b}\left(ae^{at}-be^{bt}\right)$	$\frac{s}{(s-a)(s-b)}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
$\sinh(at)$	$\frac{a}{s^2-a^2}$
$\cosh(at)$	$\frac{s}{s^2-a^2}$
$\sin(at)\sinh(at)$	$\frac{2a^2}{s^4+4a^4}$
$\frac{d^n f}{dt^n}(t)$	$s^n F(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$