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UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2011/12

BSc. II

TITLE OF PAPER	:	MATHEMATICS FOR SCIENTISTS
COURSE NUMBER	:	M215
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

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QUESTION 1

(a) Does the point $R(2,-1)$ lie on the line through $P(-1,1) and Q(5,-3)$? [4]

(b) Find the angle between $\overline{u}=(1,-2,4)$ and $\overline{v}=(1,2,4)$. [4]

(c) Use vector product to find the area of the parallelogramm spanned by the vectors $\overline{a}=(2,0,0)$ and $\overline{b}=(2,2,0)$. [5]

(d) Find the volume of parallelepiped spanned by the directed segments $\overline{OA}, \overline{OB}$ and $\overline{OC}$, if the coordinates of A, B and C are $(8,4,0),(2,6,0),(0,4,6)$, respectively. [7]

QUESTION 2

(a) If $f(x)=x^2+1$, find all numbers in the interval $(1,2)$ for which the mean value theorem is satisfied. [3]

(b) Compute

(i) $\lim\limits_{x\to 0}\dfrac{tanx}{x}$,

(ii) $\lim\limits_{x\to +\infty}\dfrac{x\ln x}{(x+1)^2}$,

(iii) $\lim\limits_{x\to +0} x\exp\left(\dfrac{1}{x}\right)$. [3,3,5]

(c) Use the quadratic approximation to compute $\sqrt{1+x}$ for small $|x|$ and estimate the error. In particular compute $\sqrt{0.164}$. [6]

QUESTION 3

(a) Find the fourth Taylor polynomial at $x_0=1$, for $f(x)=x^4$. [4]

(b) Find the partial derivatives of $f(x,y)=\sin(x^2+y)$, at $x=0, y=\pi$. [3]

(c) Use the chain rule to evaluate f'_u and f'_v if $f(x,y)=x^2+y^2$, $x=u\cos v, y=u\sin v$. [6]

(d) Verify equality of mixed derivatives theorem for

$$f(x,y)=\sin(2x+3y).$$

[3]

(e) If $f(x,y)=x^2e^{xy}$, what is df? [4]

QUESTION 4

a) For the function $f = x^2 + xy + yz + 4z$ find

(i) the gradient,

(ii) the stationary points. [2,2]

b) Find and classify all stationary points if $f(x, y) = x^3 + y^3 + 3xy$. [4]

c) Use the method of Lagrange to find the extreme value of $f(x, y) = x^2 + y^2$, subject to constraint $x + 2y = 3$. [5]

(d) A farmer who wants to create a rectangular grazing field bordering on a straight river has 600m of fencing material. If the side along the stream will not be fenced what length and width will provide the maximum grazing area? [7]

QUESTION 5

a) Compute the volume under the graph of $z = f(x, y) = xy + 1$ over the region $0 < x < 2$, $0 < y < 4$. [4]

b) Compute $\int\int_D x^2 y dx dy$ if D is the interior of the triangle with vertices $(0,0), (0,1), (1,1)$. [5]

c) Use the separation of variables to evaluate integral of $f = y^2 e^{-x}$, if $0 < x < 1$, $0 < y < 9$. [4]

d) Compute $\int\int_D \sqrt{x^2 + y^2} dx dy$ in polar coordinates, if D is a region in the first quadrant bounded by the unit circle and the coordinate axes. [7]

QUESTION 6

a) Compute $\int\int_D\int yz^2e^{xyz}\ dxdydz$, where D is a cube $0<x<1,\ \ 0<y<1,\ \ 0<z<1.$ [6]

b) Let D be the region defined by the inequalities $x^2+y^2<1,\ \ 0<z<x^2+y^2$. Pass to cylindrical coordinated to find $\int\int_D\int x^2y^2\ \ dxdydz.$ [8]

c) Separate the variables to solve the following initial value problem,

$y'=ky,\ y=y_0$ when $x=0$. [6]

QUESTION 7

a) Consider ODE

$$M(x,y)dx+N(x,y)dy=0.$$

Let M and N be homogeneous functions (HF) of the same degree in x and y. Show that $f=\frac{M}{N}$ is the HF of degree 0. [3]

b) Consider the following ODE

$$2(2x^2+y^2)dx-xydy=0$$

(i) Show that the coefficients are HF.

(ii) Solve the equation. [1,4]

c) Consider ODE

$$3x(xy-2)dx+(x^3+2y)dy=0.$$

[2,5]

(i) Test it for exactness, (ii) Solve ODE.

(d) Solve the following initial value problem,

$$y''-2y'-3y=0,$$

$$y(0)=0,\ \ y'(0)=-4.$$

[5]