UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2011/2012

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Consider the iterative schemes:

$$x_{n+1} = \frac{1}{5}\left(x_n^3 + 1\right)\ ,\quad x_{n+1} = (5x_n - 1)^{1/3},\quad x_{n+1} = \sqrt{\frac{5x_n - 1}{x_n}},\qquad n \geq 0$$

for solving the non-linear equation $x^3 - 5x + 1 = 0$ in the interval [0,1].

i. Which one is suitable for approximating the solution of the given equation? [5 Marks]

ii. Starting from $x_0 = 1$, compute the fourth approximation x_4. [5 Marks]

(b) Show that the iteration scheme

$$\alpha_{n+1} = \frac{\alpha_n^2 - a\alpha_n + a^2 + 5a}{\alpha_n + 5}$$

converges to the fixed point a quadratically (i.e order of convergence is 2) for all $a \neq -5$. [5 Marks]

(c) Show that the iterative scheme

$$x_{n+1} = \frac{(3A - x_n^2)x_n}{2A},\qquad A > 0,\ \ n \geq 0$$

converges to $\sqrt{A}$ and its rate of convergence is quadratic. [5 Marks]

QUESTION 2

2. (a) Consider the function $f(x) = x^3 + 4x^2 - 10$.

i. Show that $f(x)$ has **exactly** one root in [1, 2]. [6 Marks]

ii. By performing 4 iterations of the bisection method, show that this root lies in the interval [1.3125, 1.375]. [8 Marks]

iii. How many iterations would be required to locate this zero to a tolerance of 10^{-5}? [6 Marks]

QUESTION 3

3. (a) Convert the decimal 9.7 into its binary equivalent. [6 Marks]

   (b) Convert the binary $(0.\overline{10})_2$ into its decimal equivalent. [6 Marks]

   (c) Determine the machine representation in single precision on a 32-bit word length computer (Marc-32) for the decimal number -285.75 [8 Marks]

QUESTION 4

4. (a) Transform $\int_{-3}^{3} \frac{1}{t^2+1}\, dt$ to an integral of the form $\int_{-1}^{1} f(x)dx$ [6 Marks]

   (b) Find the coefficients below for the three-point Gaussian quadrature rule:

$$\int_{-1}^{1} f(x)dx \approx af\left(-\sqrt{\frac{3}{5}}\right) + bf(0) + cf\left(+\sqrt{\frac{3}{5}}\right)$$

[8 Marks]

   (c) Estimate the integral from (a) using this rule. Express your answer as a fraction. [6 Marks]

QUESTION 5

5. (a) Find the LU factorisation of the matrix A in which the diagonal elements of L are 1 for

$$A = \begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 2 & 2 & 0 \\ 0 & 2 & 8 & -6 \\ 0 & 0 & -6 & 10 \end{bmatrix}$$

[10 Marks]

   (b) Use the LU factorisation in (a) to find x such that

$$Ax = \begin{bmatrix} 2 \\ 0 \\ -8 \\ 16 \end{bmatrix}$$

[10 Marks]

QUESTION 6

6. (a) Consider the points $x_0 = 1$, $x_1 = 1.5$, $x_2 = 2.5$ and for a function $f(x)$, the divided differences are $f[x_2] = 5$, $f[x_1, x_2] = 15$, $f[x_0, x_1, x_2] = 35$. Use this information to construct the complete divided differences table for the given points. [8 Marks]

   (b) Consider the integral $\int_0^1 \sin\left(\frac{\pi x^2}{2}\right) dx$. Suppose we wish to integrate it numerically with an error of magnitude less than 10^{-3}. What width h is needed if we wish to use the composite Trapezoid rule? [6 marks]

   (c) Evaluate $\int_2^6 \frac{x}{1+x} dx$ using the Simpson rule with $h = 1$ and calculate the error against the exact value of the integral to four decimal places. [6 marks]

QUESTION 7

(a) Suppose that $f(-1) = 3$, $f(0) = 4$ and $f(2) = 5$. Find the Lagrange interpolating polynomial which interpolates these values, and use it to estimate $f'(0)$. [8 Marks]

(b) Convert the 32-bit floating-point number 0 1000 1010 010 0010 0011 1010 0000 0000 to its decimal equivalent. [6 marks]

(c) Given the function $f(h) = \sqrt{9-h} - 3$

   (i) find a suitable function $g(h)$ that has been reformulated to be algebraically equivalent to $f(h)$ with the aim of avoiding loss of significance error. [3 marks]

   (ii) Compare the results of calculating $f(0.0001)$ and $g(0.0001)$ using **six digits and chopping.** [3 marks]