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UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATIONS 2012

BSc. / BEd. / B.A.S.S. III

TITLE OF PAPER	:	NUMERICAL ANALYSIS I
COURSE NUMBER	:	M 311
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

1. (a) Convert the following decimal numbers to their binary equivalent

   i. 0.6 [5 Marks]

   ii. $\frac{17}{16}$ [5 Marks]

   (b) Convert the following binary numbers to their decimal equivalent

   i. $(11011.01)_2$ [5 Marks]

   ii. $(0.0\overline{0011})_2$ [5 Marks]

QUESTION 2

2. (a) Use 3 iterations of the secant method to estimate the root of

$$f(x) = \ln x - e^x + 3$$

   with starting points $x_0 = 1$ and $x_1 = 2$ [6 marks]

   (b) Let $p_2(x)$ be the quadratic polynomial interpolating $f(x)$ at $(0, f(0))$, $(h, f(h))$ and $(2h, f(2h))$.

   (i) Write down the Lagrange representation of $p_2(x)$. [7 marks]

   (ii) By integrating $p_2(x)$ between 0 and $3h$, derive the numerical integration rule that approximates $I = \int_0^{3h} f(x)\,dx$; that is, show that

$$I \approx \frac{3h}{4}\left[f(0) + 3f(2h)\right].$$

   [7 marks]

QUESTION 3

3. (a) Consider the iterative scheme

$$x_{n+1} = \frac{12}{1 + x_n} \qquad n \geq 0.$$

   i. Find the positive fixed point, s, of the scheme. [2 marks]

   ii. Prove that the scheme convergences to s for sufficiently close x_0; determine the order and corresponding asymptotic error constant for this method. [8 marks]

   (b) Given the data

x	0	1	3	2	5
$f(x)$	2	1	5	6	-183

   (i) construct a divided difference table. [5 marks]

   (ii) write down the Newton form of the interpolating polynomial. [5 marks]

QUESTION 4

4. (a) Use the two-point Gaussian Quadrature rule,

$$\int_{-1}^{1} f(x)\, dx \approx f\left(\frac{-\sqrt{3}}{3}\right) + f\left(\frac{\sqrt{3}}{3}\right),$$

to approximate the integral

$$\int_{0}^{1} x^2 e^{-x}\, dx$$

and compare your result against the exact value of the integral. [10 marks]

(b) Evaluate the integral $\int_{0}^{1} xe^{-x}\, dx$ analytically correct to four decimal places. Use the trapezoidal rule with $h = 0.2$ and the Simpson's rule with $h = 0.25$ to compute the same integral. Compare the errors. [10 marks]

QUESTION 5

5. Given the points (-2,-1),(-1,3),(0,1),(1,-1) and (2,3)

(a) Construct a forward difference table. [10 marks]

(b) Prove that the polynomial of degree ≤ 4 that goes through the points in Newton form is

$$x^3 - 3x + 1$$

[10 marks]

QUESTION 6

6. Consider the function $f(x) = x^3 + 4x^2 - 10$.

(a) Show that $f(x)$ has exactly one root in [1, 2]. [6 marks]

(b) By performing 4 iterations of the bisection method, show that this root lies in the interval [1.3125, 1.375]. [8 marks]

(c) How many iterations would be required to locate this zero to a tolerance of 10^{-5}? [6 marks]

QUESTION 7

7. Given the tridiagonal matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

determine the LU decomposition of A, and hence solve the system $Ax = b$ where $b = [2, 1, 1, 0]^T$

[20 marks]