UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2011/12

B.Sc. / B.Ed. / B.A.S.S III

TITLE OF PAPER	:	DYNAMICS II
COURSE NUMBER	:	M355
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

QUESTION 1

(a) Give the definitions and some examples of

(i) generalized coordinates,

(ii) holonomic systems,

(iii) scleronomic systems. [2,2,2]

(b) Prove that the force of gravity is conservative. [3]

(c) Prove the interchange of d and ∂ Lemma

$$\frac{d}{dt}\left(\frac{\partial \bar{r}_\gamma}{\partial q_i}\right) = \frac{\partial \dot{\bar{r}}_\gamma}{\partial q_i}.$$

[7]

(d) Let a particle of the mass m be in the field of gravity.

(i) Derive Lagrange equation and

(ii) solve it. [2,2]

QUESTION 2

(a) Two particles of masses m_1 and m_2 are connected by a light inextensible string of length l and negligible mass which passes over a frictionless pulley. Set up the Lagrangian and find the acceleration of mass m_1. [5]

(b) System has two degrees of freedon with the generalized coordinates θ and ρ. Kinetic and potential energies are as follows

$T = \frac{2}{3}Ma^2(\dot{\theta}^2 + \dot{\rho}^2 \sin^2\theta)$,

$\Pi = -Mga\cos\theta, \qquad M, g, a$ are const.

(i) Derive Lagrange equations,

(ii) Find a cyclic coordinate and thus find the constant of motion. [5,3]

(c) Let the potential energy be

$\Pi = \Pi(q, \dot{q})$. Show that

$T + \Pi = \sum_{i=1}^{n} \dot{q}_i \frac{\partial \Pi}{\partial \dot{q}_i} = const.$ in the usual notations. [7]

QUESTION 3

(a) Derive Hamilton's equations if $H = H(q,p)$. [5]

(b) For the mathematical pendulum, find

(i) generalized momentum,

(ii) Hamiltonian,

(iii) Hamilton's equations. [2,2,2]

(c) Let x and y be generalized coordinates. Given kinetic energy

$2T = M[(\dot{x} - \omega y)^2 + (\dot{y} + \omega x)^2]$ and potential energy $\Pi(x,y)$.

Find

(i) generalized momenta,

(ii) Hamiltonian,

(iii) Hamilton's equations.

[3,3,3]

QUESTION 4

a) Prove that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$. [4]

b) Use all three conditions to demonstrate that the transformation

$Q = p \quad , P = -q$ is cononical. [8]

c) Using Jacobian show that the transformation

$Q = \cos q + \sin p, \quad P = -\sin q + \cos p$ is not canonical. [3]

d) (i) Define Poisson bracket between two physical quantities.

(ii) Prove that

$[A, B + C] = [A, B] + [A, C]$. [2,3]

QUESTION 5

a) Prove that

$[q_k, p_l]_{q,p} = \delta_{kl}$, in the usual notations. [5]

b) Use Poisson brackets to show that transformation $q = \sqrt{\frac{P}{\pi\omega}}\sin(2\pi Q), \quad p = \sqrt{\frac{\omega P}{\pi}}\cos(2\pi Q)$ is canonical. [5]

c) (i) State Jacobi's identity. (JI).

(ii) Apply JI to show that if X and Y are both constants of motion then $[X, Y]$ is also constant of motion. [2,8]

QUESTION 6

a) Consider a functional

$$V[y(x)] = \int_{x_0}^{x_1} F(x, y(x), y'(x))dx,$$

subject to the boundary conditions $y(x_0) = y_0, \quad y(x_1) = y_1$. Show that if $y(x)$ is an extremal, then it satisfies Euler equation. [7]

b) Find the extremals and the extreme value of V if

$V[y(x)] = \int_1^2 (yy' + (y')^2)dx, \quad y(1) = 1, \quad y'(2) = 2.$ [5]

c) Let $F = y\sqrt{1 + (y')^2}$. Construct

(i) Euler equation,

(ii) Beltrami identity. [4,4]

QUESTION 7

a) Find extremals for the following functionals

(i) $V[y(x), z(x)] = \int_0^{\frac{\pi}{2}} (y'^2 + z'^2 + 2yz)dx;$

$y(0) = z(0) = 0; \;\; y\left(\frac{\pi}{2}\right) = z\left(\frac{\pi}{2}\right) = 1.$

(ii) $V[Y(X)] = \int_0^{\frac{\pi}{2}} (y''^2 - y^2 + x^2)dx,$

$y(0) = 1, \;\; y'(0) = y\left(\frac{\pi}{2}\right) = 0, \;\; y'\left(\frac{\pi}{2}\right) = -1.$ [6,8]

b) Find Ostrogradski's equation for the following functional

$$V[Z(x,y)] = \int\int_D \left[\left(\frac{\partial z}{\partial x}\right)^2 - \left(\frac{\partial z}{\partial y}\right)^2\right] dxdy,$$

where $z(x, y)$ is known on the boundary of region D. [6]