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UNIVERSITY OF SWAZILAND

FINAL EXAMINATION 2011/12

BSC IV

TITLE OF PAPER	:	FLUID DYNAMICS
COURSE NUMBER	:	M455
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	1. THIS PAPER CONSISTS OF SEVEN QUESTIONS. 2. ANSWER ANY FIVE QUESTIONS
SPECIAL REQUIREMENTS	:	NONE

THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

USEFUL FORMULAE

The gradient of a function $\psi(r, \theta, z)$ in cylindrical coordinates is

$$\nabla\psi = \frac{\partial \psi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\hat{\theta} + \frac{\partial \psi}{\partial z}\hat{k}$$

The divergence and curl of the vector field

$$\underline{v} = v_r\hat{r} + v_\theta\hat{\theta} + v_z\hat{k}$$

in cylindrical coordinates are

$$\nabla \cdot \underline{v} = \frac{1}{r}\left\{\frac{\partial}{\partial r}(rv_r) + \frac{\partial}{\partial \theta}(v_\theta) + \frac{\partial}{\partial z}(rv_z)\right\}$$

and

$$\nabla \times \underline{v} = \frac{1}{r}\det\begin{bmatrix} \hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ v_r & rv_\theta & v_z \end{bmatrix}$$

The divergence of a vector

$$\underline{v} = v_r\hat{r} + v_\lambda\hat{\lambda} + v_\theta\hat{\theta}$$

in spherical coordinates

$$\nabla \cdot \underline{v} = \frac{1}{r^2}\frac{\partial(r^2 v_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial v_\lambda}{\partial \lambda} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta\, v_\theta)}{\partial \theta}$$

The convective derivative and Laplacian in cylindrical coordinates are

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_z\frac{\partial}{\partial z}$$

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

Identities

$$\underline{v} \cdot \nabla\underline{v} = \nabla\left(\frac{v^2}{2}\right) - \underline{v} \times \underline{\omega}$$

$$\nabla \times (\nabla \times \underline{a}) = \nabla\nabla \cdot \underline{a} - \nabla^2\underline{a}$$

QUESTION 1

(a) Describe density of the air at a point. [4]

(b) For the Lagrangian method describe

(i) trajectory, (ii) velocity, (iii) acceleration. [2,2,2]

(c) A velocity field is specified as

$\overline{V} = ax^2\overline{i} + bxy\overline{j}$, where $a = 2/(m.s)$, $b = -4/(m.s)$, and the coordinates are measured in meters.

(i) Is the flow field one -, two-, or three- dimensional? Explain.

(ii) Calculate the velocity components at the point $(2, \frac{1}{2}, 0)$.

(iii) Develop an equation for the streamline passing through this point. [1,1,3]

(d) Derive the formula for convective derivative of the density. [5]

QUESTION 2

(a) Which of the following sets of equations represent possible two-dimensional incompressible flow cases? Explain.

(i) $u = x + y, \quad v = x - y;$

(ii) $u = x + 2y, \quad v = x^2 - y^2;$

(iii) $u = xt + 2y, \quad v = x^2 - yt^2.$ [3]

(b) The stream function for a certain incompressible flow is given as $\psi = Axy$, A is a constant.

(i) Plot several stream lines, including $\psi = 0$,

(ii) Obtain an expression for the velocity field [2,3]

(c) (i) Prove that $\overline{V} = \nabla\psi + \overline{k}$ and thus

(ii) Show $v_r = \dfrac{1}{r}\dfrac{\partial\psi}{\partial\theta}, \quad v_\theta = -\dfrac{\partial\psi}{\partial r}.$ [3,2]

(d) The stream function for a certain incompressible flow is given as $\psi(r,\theta) = -Ur\sin\theta + \dfrac{q\theta}{2\pi}$, where U represents the free stream velocity.

(i) obtain an expression for the velocity field,

(ii) find the stagnation points, and

(iii) show that $\psi = 0$ there. [3,2,2]

QUESTION 3

(a) Consider a piston-cylinder apparatus. At one instant when the piston is $L_0 = 0.15m$ away from the closed end of the cylinder, the gas density is uniform at $\rho = 18kg/m^3$ and the piston begins to move away from the closed end at $V = 12m/s$. The gas motion is one-dimensional and proportional to the distance from the closed end; it varies linearly from zero at the end to $u = V$ at the piston.

(i) Evaluate the rate of change of gas density at this instant.

(ii) Obtain an expression for the average density as a function of time.

HINT: Density ρ is independent of x. [5,6]

(b) Let $\overline{V} = \frac{m}{r^2}\overline{e}_r$ in spherical coordinates.

(i) Show that $\overline{V}$ satisfies the continuity equation for incopressible flow, except at the origin O.

(ii) Let O be lye on a smooth surface S. What is the volume flow rate through S? [4,5]

QUESTION 4

a) Evaluate the circulation of a line vortex. [3]

b) The vorticity of a certain incompressible flow is given by the following formula

$$\varpi = \begin{cases} -Ar\sin\theta, & for \quad r < a \\ 0, & for \quad r > a \end{cases}$$

Find the corresponding stream function. [12]

HINT: $\nabla + (\nabla + \overline{A}) = \nabla\nabla \cdot \overline{A} - \nabla^2\overline{A}$.

(c) A flow is respresented by the velocity field

$\overline{V} = 10x\overline{i} - 10yj + 30\overline{k}$.

Determine if the field is

(i) a possible incompressible flow,

(ii) irrational. [2,3]

QUESTION 5

a) A cylindrical container, partly filled with liquid, is rotated at a constant angular velocity ω about the vertical axis. After a short time there is no relative motion, the liquid rotates as a rigid body. Using the Euler equation determine the shape of the free surface if the radius of cylinder is R and the original surface height in the absence of rotation is h_0. [12]

b) The velocity distribution for laminar flow between fixed parallel plates is given by

$$u = u_{max}\left[1 - \left(\frac{2y}{h}\right)^2\right],$$

where h is the distance separating the plates and the origin is placed midway between the plates. Consider $\mu = 1.1 + 10^{-3} kg/ms$, $u_{max} = 0.3 m/s$, $h = 0.5 mm$. Calculate

(i) the shear stress on the upper plate and give direction,

(ii) the force on a $0.5m^2$ section of the upper plate and give its direction. [4,2]

(c) Write Navier-Stokes equation for incompressible flow. [2]

QUESTION 6

a) An incopressible flow field is given by

$\overline{V} = (Ax + By)\overline{i} - Ay\overline{j}$, where $A = 1s^{-2}$, $B = 2s^{-2}$, the coordinates are in meters.

(i) Find the acceleration of a fluid particle at point $(x, y) = (1, 2)$.

(ii) Find the pressure gradient at the same piont, if $\overline{g} = -g\overline{j}$, and fluid is water,

$\rho = 1000 kg/m^3$. [6,4]

b) The plane $y = 0$ oscilates so that its velocity is in the plane $y = 0$ and has magnitude $V \cos \omega t$, where V and ω are constants. Above the plane there is viscous incompressible fluid. Body forces are negligible.

(i) Show that

$$\frac{\partial u}{\partial t} = \gamma \frac{\partial^2 u}{\partial y^2},$$

(ii) Separate variables to show that

$$u(y,t) = Re\left\{V \exp\left(i\omega t - \sqrt{\frac{i\omega}{\gamma}} y\right)\right\}$$ [5,5]

QUESTION 7

a) Derive the mass conservation equation for incompressible flow in dimensionless form. [4]

b) (i) Define similar flow, and

(ii) Explain how the idea of similarity is used to design the experimental models. [2,2]

c) Water flows steadily up a vertieal 0.1m diameter pipe and out the nozzle, which is 0.05m in diameter, discharging to atmospheric pressure. The stream velocity at the nozzle exit must be 20m/s. The flow is frictionless. The pipe is 4m long.

(i) Write Bernoulli's equation.

(ii) find velocity at inlet,

(iii) caculate the gage pressure required at inlet. [2,3,7]