University of Swaziland

Final Examination, December 2012

BSc I, EEng I, BEd I

Title of Paper : Algebra, Trig. & Analytic Geom.

Course Number : M111

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Find the *first 4 terms* of the binomial expansion of

$$\left(\frac{a^3}{b} - \frac{4b}{a^2}\right)^{20}.$$ [6 marks]

(b) Find the *fifth term* of the biinomial expansion of

$$\left(\frac{1}{p^4} - 4p^3\right)^{-\frac{3}{2}}.$$ [4 marks]

(c) Divide

$$\frac{x^5 - 2x^4 + 3x^2 - 2x + 5}{x^2 - 3}.$$ [6 marks]

(d) Evaluate

$$\begin{vmatrix} 1 & 2 & -3 & 0 \\ 0 & 0 & 4 & 1 \\ -2 & 1 & 3 & -1 \\ 5 & -8 & 0 & 2 \end{vmatrix}.$$ [4 marks]

Question 2

(a) Solve for x.

(i) $\log_x\left(\frac{4}{25}\right) = \frac{2}{3}$ [2 marks]

(ii) $\log_3(2x - 3) = 3$ [2 marks]

(iii) $\log_7(x + 1) + \log_7(x - 8) = \log_5 5$ [5 marks]

(b)

i. Using the general equation of a circle $x^2+y^2+Ax+By+C = 0$, find the equation of the circle passing through $(5, -2), (3, 4)$ and $(-1, -8)$. [7 marks]

ii. Hence, find the radius and centre of the circle. [4 marks]

Question 3

(a) Consider the parametric equations

$$x = 1 + 2\sin\theta, \quad y = 2 + 5\cos\theta. \tag{1}$$

i. By eliminating θ, express (1) as a single equation in terms of x and y only. [5 marks]

ii. Fully *describe* the curve defined by (1) and *make a sketch*, showing all the *key features.* [5 marks]

(b) Expand $\left(1 - i\sqrt{3}\right)^6$ and leave your answer in the form $a + ib$, using

i. the binomial theorem [5 marks]

ii. de Moivre's theorem [5 marks]

Question 4

(a) Prove

i. $\tan\theta + \dfrac{\cos\theta}{1 + \sin\theta} = \sec\theta$ [5 marks]

ii. $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \tan\theta$ [5 marks]

(b) Solve

$$\begin{aligned} x - 2y + z &= -6 \\ 2x + y - z &= 7 \\ x - y - 2z &= 7 \end{aligned}$$

using Cramer's rule. [10 marks]

Question 5

(a) Given that $z = 2 + 3i$ is a root of

$$P(z) = 6z^4 - 23z^3 + 72z^2 + 21z - 26,$$

find the 3 other roots. [9 marks]

(b) Use synthetic division to work out

$$\frac{2x^5 - 2x^4 + 2x^2 - 3x + 7}{x + 2}.$$ [4 marks]

(c) Use mathematical induction to prove that

$$P(n) = 7^n - 3^n$$

is always divisible by 4 (where $n \geqslant 1$ is an integer).

[7 marks]

Question 6

(a) Solve for x

i. $4^{2-3x} = 6 \cdot 5^x$ [3 marks]

ii. $e^x + e^{-x} = \frac{10}{3}$ [5 marks]

(b) Find the value of the sum

i. $\displaystyle\sum_{n=0}^{80}(5n + 7)$ [2 marks]

ii. $\displaystyle\sum_{n=0}^{\infty} 40\left(-\frac{3}{5}\right)^n$ [4 marks]

(c) Find a solution set of

$$\cos^2\theta - \cos\theta = \sin^2\theta$$

in the interval $-\pi \leqslant \theta \leqslant \pi$. [6 marks]

Question 7

(a) Find the value(s) of x such that the numbers

$$3x^2 + x + 1, 2x^2 + x, 4x^2 - 6x + 1$$

form an arithmetic progression. [5 marks]

(b) Use mathematical induction to prove the formula

$$a_1 + a_1 r + a_1 r^2 + a_1 r^3 + \cdots + a_1 r^{n-1} = \frac{a_1(1 - r^n)}{1 - r}, \quad r \neq 1, \ n \geqslant 1$$

for the sum of the first n terms of a GP. [9 marks]

(c) In the binomial expansion of

$$\left(2A^2 + \frac{1}{\sqrt{A}}\right)^{25},$$

i. which term is independent of A? [2 marks]

ii. find this term. [4 marks]