University of Swaziland

Final Examination May 2013

BSc I, EEng I, BEd I, BASS I

Title of Paper : Introduction to Calculus

Course Number : M115

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions printed on FIVE pages.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate

i. $\lim_{x \to 1} \dfrac{x^3 - 1}{x^2 - 1}$ [5 marks]

ii. $\lim_{x \to \infty} \left(\dfrac{x^2}{x+1} - \dfrac{x^2}{x-1} \right)$ [5 marks]

(b) Find $\dfrac{\mathrm{d}f}{\mathrm{d}x}$ using the *limit definition* for

$$f(x) = 4 - \sqrt{1 - 8x}.$$ [10 marks]

Question 2

(a) Write a short essay about *stationary points*. Your discussion, aided by graphical sketches and formulas, should highlight the connection with turning points, inflexion points and relative maximum/minimum points. [9 marks]

(b) Consider the function

$$y = x^4 + 4x^3 + 5.$$

Find the

i. stationary points and classify them [5 marks]

ii. inflexion points [3 marks]

Hence make a sketch of the graph of the function. [3 marks]

Question 3

(a) Differentiate

i. $y = \left(\cosh x\right)^{x}$ [4 marks]

ii. $y = \ln\left(\dfrac{x^2 - 1}{x^2 + 1}\right)$ [4 marks]

(b) Integrate

i. $\displaystyle\int \frac{6 - 4x}{\sqrt{x^2 - 3x + 2}}\,\mathrm{d}x$ [5 marks]

ii. $\displaystyle\int_0^2 \sqrt{4 - x^2}\,\mathrm{d}x$ [7 marks]

Question 4

(a) Use the limit definition to prove the product rule

$$\frac{\mathrm{d}}{\mathrm{d}x}\{u(x)\cdot v(x)\} = v(x)\frac{\mathrm{d}u}{\mathrm{d}x} + u(x)\frac{\mathrm{d}v}{\mathrm{d}x}.$$ [8 marks]

(b) Use Leibnitz's rule to find

$$\frac{\mathrm{d}^4}{\mathrm{d}x^4}\left(x^6 \ln x\right).$$ [7 marks]

(c) Find the *exact* area of the region enclosed between $y = 8 - x^2$ and $y = 0$. [5 marks]

Question 5

(a) Given $y = \sqrt{x^2 - 2x + 2}$, find y''. [7 marks]

(b) Differentiate and simplify

$$y = \ln\left(2x^2 - 6x + 5\right) + 6\arctan(2x - 3). \quad \text{[6 marks]}$$

(c) Evaluate the integral

$$\int \frac{\mathrm{d}x}{x + x^3}\,\mathrm{d}x. \quad \text{[7 marks]}$$

Question 6

(a). You have been assigned to design a *closed* rectangular box with a square base and a capacity of 64 cubic centimetres.

i. Show that its surface area is given by

$$S(x) = 2x^2 + \frac{256}{x}. \quad \text{[3 marks]}$$

ii. Find the value of x that minimises the surface area. [7 marks]

(b) Evaluate

i. $\int 81x^3 \cos 3x\,\mathrm{d}x$ [5 marks]

ii. $\int_0^{\frac{\pi}{2}} \sin^2\theta \cos^3\theta \mathrm{d}\theta$ [5 marks]

Question 7

(a) Evaluate

i. $\lim_{x \to 0} \dfrac{\sin 2x}{\sin 5x}$ [2 marks]

ii. $\lim_{x \to 0} \dfrac{(1+6x)^{\frac{2}{3}} - 1}{x}$ [5 marks]

(b) Find the equation of the tangent to the graph of

$$y = \left(3e^{x} + e^{-x}\right)^{2}$$

at $x = 0$. [3 marks]

(c) Integrate

i. $\int x\left(3 - x^{3}\right)^{2} \mathrm{d}x$ [3 marks]

ii. $\int_{0}^{\frac{\pi}{4}} \tan^{4}\theta \sec^{4}\theta \mathrm{d}\theta$ [7 marks]