
University of Swaziland



Supplementary Examination, July 2013

BSc I, EEng I, BEd I, BASS I

Title of Paper : Introduction to Calculus

Course Number : M115

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions printed on FIVE pages.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

THIS PAPER SHOULD NOT BE OPENED UNTIL PERMISSION HAS BEEN GIVEN BY THE INVIGILATOR.

Question 1

(a) Evaluate

i. $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 2}$ [5 marks]

ii. $\lim_{x \rightarrow \infty} \left(\frac{5x^2 - 1}{5 - x^2} \right)$ [5 marks]

(b) Find $\frac{df}{dx}$ using the *limit definition* for

$$f(x) = 3 + 8\sqrt{x}. \quad [10 \text{ marks}]$$

Question 2

(a) Consider the statement:

Points on a graph where both y' and y'' vanish are called inflexion points.

Is the statement true or false? Discuss. [9 marks]

(b) Consider the function

$$y = x^3 - 75x + 5.$$

Find the

i. stationary points and classify them [5 marks]

ii. inflexion points [3 marks]

Hence make a sketch of the graph of the function.

[3 marks]

Question 3

(a) Differentiate

i. $y = (x^2)^{x^2}$ [4 marks]

ii. $y = \frac{x^2 - 1}{x^2 + 1}$ [4 marks]

(b) Integrate

i. $\int x\sqrt{2-x} dx$ [5 marks]

ii. $\int_0^2 \frac{dx}{9+x^2}$ [7 marks]

Question 4

(a) Use the limit definition to prove the product rule

$$\frac{d}{dx} \{ \sin x \} = \cos x. \quad [7 \text{ marks}]$$

(b) Use Leibnitz's rule to find

$$\frac{d^4}{dx^4} (x^3 \ln x). \quad [7 \text{ marks}]$$

(c) Find the *exact* area of the region enclosed between
 $y = 7 - x^2$ and $y = 1 - x$. [6 marks]

Question 5

(a) Given $y = \sqrt{x^2 + 2}$, find y'' . [7 marks]

(b) Differentiate and simplify

$$y = \ln(x^2 - 6x + 9) + \frac{2}{3 - x}. \quad [6 \text{ marks}]$$

(c) Evaluate the integral

$$\int \frac{dx}{x^2 + x^3} dx. \quad [7 \text{ marks}]$$

Question 6

(a) Find the largest rectangle that can be constructed inside the ellipse $x^2 + 2y^2 = 100$. [10 marks]

(b) Evaluate

i. $\int 32x^2 \sin 2x dx$ [5 marks]

ii. $\int_0^{\frac{\pi}{2}} \sin^5 \theta d\theta$ [5 marks]

Question 7

(a) Evaluate

i. $\lim_{x \rightarrow 0} \frac{\sin^2 x}{1 - \cos x}$ [4 marks]

ii. $\lim_{x \rightarrow 0} \frac{1 - \sqrt{1 + 8x}}{x}$ [5 marks]

(b) Find the equation of the normal to the graph of

$$y = x \sin x - \cos x$$

at $x = \frac{\pi}{2}$. [3 marks]

(c) Integrate

i. $\int \left(\frac{4}{x} - 6e^{-3x} + 3x^2 \right) dx$ [3 marks]

ii. $\int_0^{\frac{\pi}{4}} \sec^4 \theta d\theta$ [5 marks]
