University of Swaziland

Final Examination, 2012/2013

BSc II, Bass II, BEd II

Title of Paper : Calculus I

Course Number : M211

Time Allowed : Three (3) hours

Instructions :

1. This paper consists of SEVEN questions.
2. Each question is worth 20%.
3. Answer ANY FIVE questions.
4. Show all your working.

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QUESTION 1

1.1 Find the absolute maximum and absolute minimum values of the function

$$f(x) = xe^{-x^2/8}$$

on the interval $[-1, 4]$. [6]

1.2 Let

$$f(x) = \frac{\ln x}{\sqrt{x}}$$

1.2.1 What is the domain of f? [1]

1.2.2 Find the critical points of f. [4]

1.2.3 Determine the intervals where f is increasing and where f is decreasing. [3]

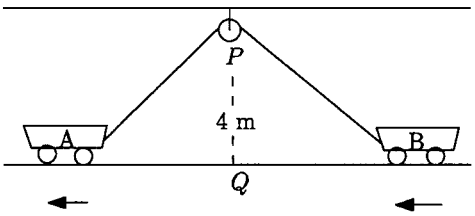
1.2.4 Determine all local extrema of f. [1]

1.2.5 Determine the intervals where f is concave up and where f is concave down. [4]

1.2.6 Find all inflection points of f. [1]

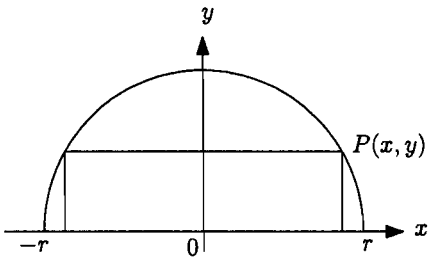
QUESTION 2

2.1 Two rail carts A and B are connected by a rope 12 m long that passes over a pulley P (see figure below).



The point Q is on the rail track 4 m directly below P and between the carts. Cart A is being pulled away from Q at a speed of 0.5 m/s. How fast is cart B moving toward Q at the instant when cart A is 3 m from Q? [10]

2.2 Show that the area of the largest rectangle that can be inscribed in a semicircle of radius r is r^2. (**Hint:** use the picture below). [10]



QUESTION 3

Evaluate each of the following limits.

3.1 $\lim_{\theta \to \pi/2} \dfrac{1 - \sin\theta}{1 + \cos 2\theta}$ [5]

3.2 $\lim_{x \to \infty} [\ln 2x - \ln(x-1)]$ [5]

3.3 $\lim_{x \to \infty} (\ln x)^{1/x}$ [5]

3.4 $\lim_{x \to \infty} x^3 e^{-x}$ [5]

QUESTION 4

4.1 The base of a solid is the region between the curve $y = 2\sqrt{\sin x}$ and the interval $[0, \pi]$ on the x-axis. The cross sections perpendicular to the x-axis are equilateral triangles with bases running from the x-axis to the curve. Find the volume of the solid. [10]

4.2 The region bounded by the curve $y = \sqrt{x}$ and the lines $y = 2$ and $x = 0$ is revolved about the line $x = 4$ to generate a solid. Find the volume of the solid. [10]

QUESTION 5

5.1 Find the length of the curve with parametric equations $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq \pi$. [10]

5.2 Use the method of cylindrical shells to find the volume of the solid generated by revolving, about the y-axis, the region bounded the curve $y = e^{-x^2}$ and the lines $y = 0$, $x = 0$ and $x = 1$. [10]

QUESTION 6

6.1 Show that the p-series $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$. [10]

6.2 Let

$$f(x) = (1 - x)^{-2}$$

Find the Maclaurin series for $f(x)$ and find the associated radius of convergence. [10]

QUESTION 7

7.1 Let $a_n = \dfrac{2n}{3n+1}$.

7.1.1 Determine whether or not $\{a_n\}$ is convergent. If it is convergent, find its limit. [2]

7.1.2 Determine whether or not $\sum_{n=1}^{\infty}$ is convergent. [3]

7.2 Use the Integral Test to determine whether the series

$$\sum_{n=1}^{\infty} \frac{1}{n^4}$$

converges or diverges. [5]

7.3 Find the radius of convergence and the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}.$$

[10]

END OF EXAMINATION PAPER